# Vahab Hassani et al. "Fundamentals"

Handbook of Heating, Ventilation, and Air Conditioning Ed. Jan F. Kreider Boca Raton, CRC Press LLC. 2001



FIGURE 2.1.38 Fluid temperature variation for (a) parallel-flow configuration, and (b) counter-flow configuration.

The heat-transfer rate between a hot and a cold fluid can be written as

$$\dot{Q} = UA \ \Delta T_m, \tag{2.1.86}$$

where U is the overall heat-transfer coefficient (and is assumed to be constant over the whole surface area of the heat exchanger) and  $\Delta T_{\rm m}$  is an appropriate mean temperature difference to be defined later. In fact, the overall heat-transfer coefficient is not the same for all locations in the heat exchanger, and its local value depends on the local fluid temperatures as was shown in Example 2.1.8. For most engineering applications, designers of heat exchangers are usually interested in the *overall* average heat-transfer coefficient. Common practice is to calculate the overall heat-transfer coefficient based on some kind of mean fluid temperatures.

Expanding on the definition of thermal resistance described earlier, the heat transfer in a heat exchanger can be expressed as

$$\dot{Q} = UA \quad \Delta T_m = \frac{\Delta T_m}{\sum_{i=1}^n R_i}, \tag{2.1.87}$$

Overall Heat-Transfer Coefficient (W/m <sup>2</sup> K)
2,200-3,300
960-1,650
55–165
165–330
275-1,100
550-3,300
25-275
55–165
825-1,510
110-330
220-550
250-660
440-830
830-1,380
275-830
275-830
330-500
200-330
550-1,100
220-420
110–330
55–220
55-330
170-300

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where  $\Sigma R_i$  represents the total thermal resistance to heat transfer between fluid streams in the heat exchanger. For example, consider the simple case of heat transfer inside a shell-and-tube heat exchanger, where a hot fluid at  $T_h$  is flowing inside a steel tube with inside radius  $r_i$  and outside radius  $r_o$  as shown in Figure 2.1.14. The cold fluid at  $T_c$  is flowing in the shell side over the steel tube, where the convection heat-transfer coefficient between the cold fluid and the exterior of steel tube is  $h_o$ . For this case, the total resistance to heat transfer can be written as

$$\frac{1}{UA} = \sum_{i=1}^{5} R_i = \frac{1}{2\pi r_i L h_i} + R_{f,i} + \frac{\ln \left( r_o / r_i \right)}{2\pi k_p L} + R_{f,o} + \frac{1}{2\pi r_o L h_o}, \qquad (2.1.88)$$

where L is the length of the heat exchanger,  $k_p$  is the thermal conductivity of steel, and  $h_i$  and  $h_o$  are the convection heat-transfer coefficients of hot and cold fluid sides, respectively. Terms  $R_{f,i}$  and  $R_{f,o}$  represent the fouling resistances on the cold and hot heat-transfer surfaces. The overall heat-transfer coefficient can be based on either the hot surface area (in this case,  $A_i = 2\pi r_i L$ ) or on the cold surface area ( $A_o = 2\pi r_o L$ ). Therefore, the numerical value of U will depend on the area selected; however, it is always true that  $UA \equiv U_i A_i \equiv U_o A_o$ .

Table 2.1.11 gives some typical values of overall heat-transfer coefficients that are useful in preliminary system analysis and design. For all but the simplest heat exchangers, designing the best heat exchanger for a given application involves using a model that accurately sums the temperature difference and the resistance over the entire surface of the heat exchanger. Most engineers use sophisticated computer models for

designing heat exchangers. These computer models incorporate the most accurate algorithms for a myriad of applications. A reasonably good estimate of heat-exchanger performance can be calculated by hand by using one of various readily available handbooks (e.g., *Handbook of Heat Exchanger Design* [1983]).

The other important term in Eq. (2.1.86) for calculating the heat-transfer rate is the mean temperature difference  $\Delta T_{\rm m}$ . The mean temperature difference for a heat exchanger depends on its flow configuration and the degree of fluid mixing in each flow stream.

For a simple single-pass heat exchanger with various temperature profiles (e.g., parallel flow, counter flow, and constant surface temperature), the mean temperature of Eq. (2.1.87) can be calculated from

$$\Delta T_m = \frac{\Delta T_i - \Delta T_o}{\ln\left(\Delta T_i / \Delta T_o\right)},\tag{2.1.89}$$

where  $\Delta T_i$  represents the greatest temperature difference between the fluids and  $\Delta T_o$  represents the least temperature difference, and only if the following assumptions hold:

- 1. U is constant over the entire heat exchanger.
- 2. The flow of fluids inside the heat exchanger is in steady-state mode.
- 3. The specific heat of each fluid is constant over the entire length of the heat exchanger.
- 4. Heat losses from the heat exchanger are minimal.

The mean temperature difference  $\Delta T_{\rm m}$  given by Eq. (2.1.89) is known as the logarithmic mean temperature difference (LMTD).

#### Example 2.1.11

Lubricating oil from a building standby generator at initial temperature of 115°C and flow rate of 2 kg/s is to be cooled to 70°C in a shell-and-tube heat exchanger. Cold water at a flow rate of 2 kg/s and initial temperature of 20°C is used as the cooling fluid in the heat exchanger. Calculate the heat-exchanger area required by employing first a counter-flow and then a parallel-flow heat-exchanger arrangement. The overall heat-transfer coefficient is U = 900 W/m<sup>2</sup> K, and the specific heat of the oil is  $c_{p,h} = 2.5$  kJ/kg K.

#### Solution:

First, we use Eq. (2.1.85) to calculate the water outlet temperature. The specific heat of water can be assumed to be constant over the temperature range of interest, and it is  $c_{p,c} = 4.182 \text{ kJ/kg K}$ .

$$\dot{m}_c c_{p,c} \left( T_{co} - T_{ci} \right) = \dot{m}_h c_{p,h} \left( T_{hi} - T_{ho} \right) \quad \text{(first law)}$$

or

$$T_{co} = \frac{m_h c_{p,h}}{\dot{m}_c c_{p,c}} \left( T_{hi} - T_{ho} \right) + T_{ci}.$$

$$T_{co} = \frac{2 \text{ kg/s } (2.5 \text{ kJ/kg K})}{2 \text{ kg/s } (4.182 \text{ kJ/kg K})} (115^{\circ}\text{C} - 70^{\circ}\text{C}) + 20^{\circ}\text{C}.$$
$$T_{co} = 46.9^{\circ}\text{C}.$$



**FIGURE 2.1.39** Temperature differences of Example 2.1.11 for (a) counter-flow arrangement, and (b) parallel-flow arrangement.

The total heat transferred from hot fluid to the cold fluid is

$$\dot{Q} = \dot{m}_c \ c_{p,c} (T_{co} - T_{ci}) = 2 \ \text{kg/s} (4.182 \ \text{kJ/kg K}) (46.9^{\circ}\text{C} - 20\text{C}) = 225 \ \text{kJ/s}.$$

For a *counter-flow* arrangement, the temperature differences are shown in Figure 2.1.39(a). The greatest temperature difference is  $\Delta T_i = 68.1^{\circ}$ C, and the least temperature difference is  $\Delta T_o = 50^{\circ}$ C. Using Eq. (2.1.89), the mean temperature can be calculated as

$$\Delta T_m = \frac{\Delta T_i - \Delta T_o}{\ln \left( \Delta T_i / \Delta T_o \right)} = \frac{68.1^{\circ} \text{C} - 50^{\circ} \text{C}}{\ln \left( 68.1 / 50 \right)} = 58.58^{\circ} \text{C} .$$

The heat-exchanger surface area for counter-flow arrangement can be obtained from Eq. (2.1.86):

$$\dot{Q} = UA \ \Delta T_m$$
,

$$A = \frac{\dot{Q}}{U \ \Delta T_m} = \frac{225 \times 10^3 \text{ J/s}}{900 \text{ W/m}^2 \text{ K} \times 58.58^{\circ}\text{C}} = 4.27 \text{ m}^2.$$

A similar procedure can be followed for the *parallel-flow* arrangement. The water outlet temperature  $T_{co}$  and the total heat transfer calculated earlier still hold for this arrangement. However, the temperature differences are as shown in Figure 2.1.39(b). The mean temperature for this arrangement is

$$\Delta T_m = \frac{\left(\Delta T_i - \Delta T_o\right)}{\ln\left(\Delta T_i / \Delta T_o\right)} = \frac{95^{\circ}\text{C} - 23.1^{\circ}\text{C}}{\ln\left(95/23.1\right)} = 50.84^{\circ}\text{C},$$

and the area required is

$$A = \frac{\dot{Q}}{U \ \Delta T_m} = \frac{225 \times 10^3 \text{ J/s}}{900 \text{ W/m}^2 \text{ K} \times 50.84^{\circ}\text{C}} = 4.92 \text{ m}^2.$$

Therefore, the heat-exchanger surface area required for parallel flow is more than that required for the counter-flow arrangement if all the other conditions are assumed to be the same. Consequently, whenever possible, it is advantageous to use the counter-flow arrangement because it will require less heat-exchanger surface area to accomplish the same job. In addition, as seen from Figure 2.1.39, with the counter-flow arrangement, the outlet temperature of the cooling fluid may be raised much closer to the inlet temperature of the hot fluid.

The LMTD expression presented by Eq. (2.1.89) does not hold for more complex flow configurations such as cross flow or multipass flows. To extend the LMTD definition to such configurations, a correction factor is defined as

$$F = \frac{\Delta T_m}{\Delta T_{m,cF}},\tag{2.1.90}$$

where  $\Delta T_{m,cF}$  is calculated from Eq. (2.1.89) for a counter-flow configuration. Bowman et al. [1940] provide charts for calculating the correction factor *F* for various flow configurations of heat exchangers. A sample of their charts is shown in Figure 2.1.40 for one fluid mixed and the other fluid unmixed. The term *unmixed* means that a fluid stream passes through the heat exchanger in separated flow channels or passages with no fluid mixing between adjacent flow passages. Note that the correction factor *F* in Figure 2.1.40 is a function of two dimensionless parameters *Z* and *P* defined as

$$Z = \frac{T_{hi} - T_{ho}}{T_{co} - T_{ci}} \equiv \frac{C_c}{C_h}$$
(2.1.91)

and

$$P = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}},$$
(2.1.92)

where the term Z is the ratio of the capacity rates of the cold and hot streams, and the term P is referred to as the temperature effectiveness of the cold stream. Kays and London [1984] provide a comprehensive representation of F charts.

or



**FIGURE 2.1.40** Correction factor for counter-flow LMTD for cross-flow heat exchangers with the fluid on the shell side mixed, the other fluid unmixed, and one pass through tube. Data extracted from "Mean Temperature Difference in Design" by Bowman et al. (1940) in *ASME Proceedings*, with permission of the publisher, The American Society of American Engineers.

#### Heat-Exchanger Design Methods

Heat-exchanger designers usually use two well-known methods for calculating the heat-transfer rate between fluid streams—the UA-LMTD and the effectiveness-NTU (number of heat-transfer units) methods.

#### UA-LMTD Method

In this method, the relationship between the total heat-transfer rate, the heat-transfer area, and the inlet and outlet temperatures of the two streams is obtained from Eqs. (2.1.87) and (2.1.90). Substituting Eq. (2.1.90) in Eq. (2.1.87) gives

$$\dot{Q} = F \times T_{m cF} \times UA. \tag{2.1.93}$$

For a given heat-exchanger configuration, one can calculate UA by identifying heat-transfer resistances and summing them as in Eq. (2.1.88), calculating  $\Delta T_{m,cF}$  from Eq. (2.1.89), reading the *F* value from an appropriate chart, and substituting them in Eq. (2.1.93) to find the heat-transfer rate. The UA-LMTD method is most suitable when the fluid inlet and outlet temperatures are known or can be determined readily from an energy-balance expression similar to Eq. (2.1.85). There may be situations where only inlet temperatures are known. In these cases, using the LMTD method will require an iterative procedure. However, an alternative is to use the effectiveness-NTU method described in the following section.

## **Effectiveness-NTU Method**

In this method, the capacity rates of both hot and cold fluids are used to analyze the heat-exchanger performance. We will first define two dimensionless groups that are used in this method—the number of heat-transfer units (NTU) and the heat exchanger effectiveness  $\varepsilon$ 

The NTU is defined as

$$NTU = \frac{UA}{C_{\min}},$$
(2.1.94)

where  $C_{\min}$  represents the smaller of the two capacity rates  $C_c$  and  $C_h$ . NTU is the ratio of the heat-transfer rate per degree of mean temperature-difference between the fluids, Eq. (2.1.86), to the heat-transfer rate per degree of temperature change for the fluid of minimum heat-capacity rate. NTU is a measure of the physical size of the heat exchanger: the larger the value of the NTU, the closer the heat exchanger approaches its thermodynamic limit.

The heat-exchanger effectiveness is defined as the ratio between the actual heat-transfer rate  $\dot{Q}$  and the maximum possible rate of heat that thermodynamically can be exchanged between the two fluid streams. The actual heat-transfer rate can be obtained from Eq. (2.1.85). To obtain the maximum heat-transfer rate, one can assume a counter-flow heat exchanger with infinite surface area, where one fluid undergoes a temperature change equal to the maximum temperature-difference available,  $\Delta T_{\text{max}} = T_{\text{hi}} - T_{\text{ci}}$ . The calculation of  $\dot{Q}_{\text{max}}$  is based on the fluid having the smaller capacity rate  $C_{\text{min}}$ , because of the limitations imposed by the Second Law of Thermodynamics (see Bejan [1993] for more detail). Therefore,

$$\dot{Q}_{\max} = C_{\min} \left( T_{hi} - T_{ci} \right),$$
 (2.1.95)

where Eq. (2.1.95) is not limited to counter-flow heat exchangers and can be applied equally to other configurations. Therefore, the effectiveness can be expressed as

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c \left(T_{co} - T_{ci}\right)}{C_{\min} \left(T_{hi} - T_{ci}\right)},\tag{2.1.96}$$

or

$$\varepsilon = \frac{C_h \left( T_{hi} - T_{ho} \right)}{C_{\min} \left( T_{hi} - T_{ci} \right)}.$$
(2.1.97)

Knowing the effectiveness of a heat exchanger, one can calculate the actual rate of heat transfer by using Eq. (2.1.96) or from

$$\dot{Q} = \varepsilon C_{\min} \left( T_{hi} - T_{ci} \right). \tag{2.1.98}$$

Expressions for the effectiveness of heat exchangers with various flow configurations have been developed and are given in heat-transfer texts (e.g., Bejan [1993], Kreith and Bohn [1993]). For example, the effectiveness of a counter-flow heat exchanger is given by

$$\varepsilon = \frac{1 - \exp\left[-\text{NTU}\left(1 - R\right)\right]}{1 - \text{R}\exp\left[-\text{NTU}\left(1 - R\right)\right]},$$
(2.1.99)

where  $R = C_{\min}/C_{\max}$ . The effectiveness of a parallel-flow heat exchanger is given by

$$\varepsilon = \frac{1 - \exp\left[-\text{NTU}\left(1+R\right)\right]}{1+R}.$$
(2.1.100)

The effectiveness for heat exchangers of various flow configurations has been evaluated by Kays and London [1984] and is presented in a graph format similar to the one shown in Figure 2.1.41. In this figure, the heat-exchanger effectiveness has been plotted in terms of NTU and *R*. Note that for an evaporator and a condenser, R = 0 because the fluid remains at a constant temperature during the phase change.

The two design-and-analysis methods just described are equivalent, and both can be equally employed for designing heat exchangers. However, the NTU method is preferred for rating problems where at least one exit temperature is unknown. If all inlet and outlet temperatures are known, the UA-LMTD method does not require an iterative procedure and is the preferred method.



FIGURE 2.1.41 Heat-transfer effectiveness as a function of number of heat-transfer units and capacity-rate ratio; cross-flow exchanger with one fluid mixed. Data extracted from Compact Heat Exchangers, 3rd ed., by W.M. Kays and A.L. London (1984), with permission of the author.

# Nomenclature

- A heat-transfer surface area, m<sup>2</sup> (ft<sup>2</sup>)
- speed of propagation of radiation energy, m/s (ft/s) С
- $C_{c}$ capacity rate of cold fluid equal to  $\dot{m}_{c}(c_{p})_{c}$
- $C_{\rm h}$ capacity rate of hot fluid equal to  $\dot{m}_{\rm h}(c_{\rm p})_{\rm h}$
- $C_0$ speed of light in a vacuum,  $3 \times 10^8$  m/s (9.84 × 10<sup>8</sup> ft/s)
- specific heat, kJ/kg K (Btu/lbm R)
- $c_{
  m p} D_{
  m H}$ hydraulic diameter, defined by Eq. (2.1.50)
- total energy of a thermodynamic system, kJ (Btu) Ε
- internal energy of a thermodynamic system, kJ/kg (Btu/lbm) е
- E total emissive power of a blackbody, W/m<sup>2</sup> (Btu/h ft<sup>2</sup>)
- spectral blackbody emissive power, given by Eq. (2.1.56), W/m<sup>3</sup> (Btu/h ft<sup>2</sup> $\mu$ )  $E_{\rm b\lambda}$
- f dimensionless friction factor, see Eq. (2.1.77)
- $F_{1-2}$ radiation shape factor between surfaces 1 and 2
- gravitational acceleration, 9.81 m/s<sup>2</sup> (32.2 ft/s<sup>2</sup>) g
- Newton constant, equal to 32.2 ft × lbm/(lbf × s<sup>2</sup>), or  $10^3$  kg × m<sup>2</sup>/(kJ × s<sup>2</sup>) gc
- enthalpy, kJ/kg (Btu/lbm), or convection heat-transfer coefficient, W/m<sup>2</sup> K (Btu/h ft<sup>2</sup> °F) h

- $h_{\rm f}$ enthalpy of saturated liquid, kJ/kg (Btu/lbm)
- $h_{g}$ enthalpy of saturated vapor, kJ/kg (Btu/lbm)
- k thermal conductivity, W/m K (Btu/h ft °F)
- $k_{\rm b}$ pressure-loss coefficient due to bends and fittings, see Eq. (2.1.79)
- k, pressure-loss coefficient due to sudden contraction of pipe, see Eq. (2.1.81)
- k, pressure-loss coefficient due to sudden enlargement of pipe, see Eq. (2.1.80)
- conductivity of steel plate, W/m K (Btu/h ft °F)  $k_{\rm p}$
- length of the circular cylinder or heat exchanger, m (ft) Ē
- mass of a system, kg (lbm) т
- mass-flow rate of working fluid, kg/s (lbm/s) 'n
- index of refraction of a medium n
- Nusselt number, defined by Eq. (2.1.33) Nu
- pressure, N/m<sup>2</sup> (lbf/in<sup>2</sup>) Ð
- Pr Prandtl number, equal to  $v/\alpha$
- heat absorbed from a high-temperature reservoir, kJ (Btu)  $Q_{\rm H}$
- $\begin{array}{c} Q_{\rm L} \\ \dot{Q} \\ q''_{\cdot} \end{array}$ heat rejected to a low-temperature reservoir, kJ (Btu)
- rate of energy (heat) transfer, W (Btu/h)
- heat flux
- ġ, heat-transfer rate by radiation, W (Btu/h)
- R thermal resistance, defined by Eq. (2.1.30), m<sup>2</sup> K/W (h ft<sup>2o</sup>F/Btu)
- Rayleigh number, defined by Eq. (2.1.34) Ra
- Re Reynolds number, equal to  $uD_H/v$
- $R_{\rm f.i}$ fouling resistance of cold (inner) heat transfer surface, m<sup>2</sup> K/W (h ft<sup>2</sup>°F/Btu)
- fouling resistance of hot (outter) heat transfer surface, m<sup>2</sup> K/W (h ft<sup>2</sup>°F/Btu)  $R_{\rm f.o}$
- inner radius of cylinder, m (ft) r
- outer radius of cylinder, m (ft)  $r_{0}$
- S entropy, kJ/K (Btu/R)
- entropy of saturated liquid, kJ/kg K (Btu/lbm R)  $s_{\rm f}$
- entropy of saturated vapor, kJ/kg K (Btu/lbm R) s<sub>g</sub> T
- temperature, °C (°F)
- Тн temperature of higher-temperature reservoir, K (R)
- $T_{\rm L}$ temperature of low-temperature reservoir, K (R)
- overall heat transfer coefficient, given by Eq. (2.1.88), W/m<sup>2</sup>K (Btu/h ft<sup>2o</sup>F) U
- velocity, m/s (ft/s) и
- specific volume (volume per unit mass), m<sup>3</sup>/kg (ft<sup>3</sup>/lbm) ν
- work or energy, kJ (Btu) w
- rate of work (done on the surroundings), kJ/s (Btu/h) ŵ
- actual work of a system, kJ (Btu) Wact
- frictional pressure losses, kJ/s (Btu/h)  $\dot{W}_{\rm f}$
- work done under an isentropic process, kJ (Btu)  $W_{isent}$
- work of a system under reversible process, kJ (Btu)  $W_{\rm rev}$
- distance along x axis, m (ft) х
- distance along  $\gamma$  axis, m (ft) y
- distance along z axis, or elevation of a thermodynamic system, m (ft) z

# **Greek Symbols**

- α thermal diffusivity, m<sup>2</sup>/s (ft<sup>2</sup>/h), or absorptivity of a surface
- $\bar{\alpha}$ average absorptivity of a surface
- thermal expansion coefficient, K<sup>-1</sup> (R<sup>-1</sup>) β
- δ velocity boundary-layer thickness, mm (in.)
- $\delta_{\rm T}$ temperature boundary-layer thickness, mm (in.)

- $\Delta T_{\rm i}$  equal  $T_{\rm ho} T_{\rm ci}$  for counter-flow, and  $T_{\rm hi} T_{\rm ci}$  for parallel-flow heat exchanger, see Figure 2.1.38
- $\Delta T_{o}$  equal to  $T_{hi} T_{co}$  for counter-flow, and  $T_{ho} T_{co}$  for parallel-flow heat exchanger, see Figure 2.1.38
- ε emissivity of a surface
- $\bar{\epsilon}$  average absorptivity of a surface
- $\eta_m$  mechanical efficiency, defined by Eq. (2.1.17)
- $\eta_s$  isentropic efficiency, defined by Eq. (2.1.18)
- $\eta_r$  relative efficiency, defined by Eq. (2.1.19)
- $\eta_{rev}$  energy conversion efficiency for a reversible system
- $\eta_{\rm T}$  thermal efficiency, defined by Eq. (2.1.20)
- $\lambda$  wavelength of radiation energy, m (ft)
- $\mu$  viscosity of fluid, Ns/m<sup>2</sup>
- v kinematic viscosity,  $m^2/s$  (ft<sup>2</sup>/h), or frequency,  $s^{-1}$
- ρ reflectivity of a surface, or density (mass per unit volume) kg/m<sup>3</sup> (lbm/ft<sup>3</sup>)
- $\sigma$  Stefan-Boltzmann constant, Wm<sup>-2</sup>K<sup>-4</sup> (Btu/h ft<sup>2</sup>R<sup>4</sup>)
- $\tau$  transmissivity of a surface, or frictional shear stress, N/m<sup>2</sup> (lbf/ft<sup>2</sup>)
- υ thermal radiation source frequency, s<sup>-1</sup>

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# 2.2 Psychrometrics and Comfort

# T. Agami Reddy

A large fraction of the energy used in buildings goes toward maintaining indoor thermal comfort conditions for the occupants. This section presents the basic thermodynamic relations of the important air conditioning processes and illustrates their application to the analysis of energy flows in one-zone spaces. A discussion of factors affecting human thermal comfort is also provided.

# 2.2.1 Atmospheric Composition and Pressure

Atmospheric air is not only a mixture of several gases, water vapor, and numerous pollutants; it also varies considerably from location to location. The composition of dry air is relatively constant and varies slightly with time, location, and altitude. The standard composition of dry air has been specified by the International Joint Committee on Psychrometric Data in 1949 as shown in Table 2.2.1.

TABLE 2.2.1	Composition of Dry	Air
Constituent	Molecular Mass	Volume Fraction
Oxygen	32.000	0.2095
Nitrogen	28.016	0.7809
Argon	39.944	0.0093
Carbon Dioxide	44.010	0.0003

The ASHRAE Fundamentals Handbook (1997) gives the following definition of the U.S. standard atmosphere:

- (a) Acceleration due to gravity is constant at  $32.174 \text{ ft/sec}^2 (9.807 \text{ m/s}^2)$ .
- (b) Temperature at sea level is 59.0°F (15°C or 288.1 K).
- (c) Pressure at sea level is 29.921 inches of mercury (101.039 kPa).
- (d) The atmosphere consists of dry air, which behaves as a perfect gas.

The total atmospheric pressure at different altitudes is given in standard gas tables. For altitudes up to 60,000 ft (18,291 m) the following equation can be used:

$$P = a + b \cdot H \tag{2.2.1}$$

where the constants a and b are given in the following table, H is the elevation above sea level, and pressure P is in inches of Hg or in kPa.

	$\rm H \leq 4000~f$	t (1220 m)	H > 4000 f	t (1220 m)
Constant	IP	SI	IP	SI
a	29.92	101.325	29.42	99.436
b	-0.001025	-0.01153	-0.0009	-0.010

In HVAC applications, the mixture of various constituents that compose dry air is considered to be a single gas. The molecular mass of dry air can be assumed to be 28.965.

# 2.2.2 Thermodynamic Properties of Moist Air

A *property* is any attribute or characteristic of matter that can be observed or evaluated quantitatively. Thermodynamic properties, i.e., those concerned with energy and its transformation, of primary interest to HVAC are described below.

A. Temperature t of a substance indicates its thermal state and its ability to exchange energy with a substance in contact with it. Reference points are the freezing point of water (0°C in the Celsius scale and 32°F in the Fahrenheit scale) and the boiling point of water (100°C in the Celsius scale and 212°F in the Fahrenheit scale). Often the absolute temperature scale T is more relevant:

SI units : Kelvin scale:  $T = t + 273.15^{\circ}$  (K) with t in °C IP units : Rankine scale:  $T = t + 459.67^{\circ}$  (°R) with t in °F.

- B. *Pressure* p is the normal or perpendicular force exerted by a fluid per unit area against which the force is exerted. Absolute pressure is the measure of pressure above zero; gauge pressure is measured above existing atmospheric pressure. The unit of pressure is pound per square inch (psi) or Pascal (Pa). Standard atmospheric pressure is 101.325 kPa or 14.696 psi.
- C. Density  $\rho$  of a fluid is its mass per unit volume. It is more common to use its reciprocal, the *specific volume* v, the volume occupied per unit mass. The density of air at standard atmospheric pressure and 77°F (25°C) is approximately 0.075 lbm/ft<sup>3</sup> (1.2 kg/m<sup>3</sup>). Density and specific volume of a vapor or gas are affected by both pressure and temperature. Tables are used to determine both; in some cases, they can be calculated from basic thermodynamic property relations. The density of liquids is usually assumed to be a function of temperature only.
- D. *Internal energy* u refers to the energy possessed by a substance due to the motion and/or position of the molecules. This form of energy consists of two parts: the internal kinetic energy due to the velocity of the molecules, and the internal potential energy due to the attractive forces between molecules. Changes in the average velocity of molecules are indicated by temperature changes of a substance.
- E. *Enthalpy* h is another important property defined as (u+pv) where u is the internal energy (itself a property) and (pv) is the *flow work*, i.e., the work done on the fluid to force it into a control volume. Enthalpy values are based on a specified datum temperature value.
- F. *Specific heat* c of a substance is the quantity of energy required to raise the temperature of a unit mass by 1°R or 1 K. For gases, one distinguishes between two cases:

specific heat at constant volume  $c_v = \left(\frac{\delta u}{\delta T}\right)_v$  and

specific heat at constant pressure  $c_p = \left(\frac{\delta h}{\delta T}\right)_p$ .

The former is appropriate in air conditioning because the processes occur at constant pressure. Approximately,  $c_p = 0.24$  Btu/lbm °F (1.00 kJ/kg K) for dry air, 1.0 Btu/lbm °F (4.19 kJ/kg K) for liquid water, and 0.444 Btu/lbm °F (1.86 kJ/kg K) for water vapor.

One needs to distinguish between *extensive* properties and *intensive* properties. While extensive properties are those, such as volume V (ft<sup>3</sup> or m<sup>3</sup>) or enthalpy H (Btu or kJ), that depend on the mass of the substance, intensive properties, such as temperature and pressure, do not. Our notation is to use lowercase symbols for intensive properties, i.e., properties per unit mass. Thus, v would denote the specific volume, while u and h denote the internal energy and enthalpy per unit mass, respectively. The use of the term *specific* (for example, specific enthalpy) is recommended in order to avoid ambiguity.

- G. *Phase* is a quantity of matter homogeneous throughout in chemical composition and physical structure. A pure substance is one that is uniform and invariable in chemical composition. Thus, a pure substance may exist in more than one phase, such as a mixture of liquid water and water vapor (steam). On the other hand, a mixture of gases (such as air) is not a pure substance. However, if no change of phase is involved (as in most HVAC processes), air can be assumed to be a pure substance.
- H. *Ideal gas law.* The ideal gas law is a relationship between the pressure, specific volume, and absolute temperature of the substance:

$$pv = RT \tag{2.2.2a}$$

- where  $p = absolute pressure, lbf/ft^3 (Pa)$ 
  - v = specific volume, ft<sup>3</sup>/lbm (m<sup>3</sup>/kg)
  - T = absolute temperature, °R (K), and
  - R = gas constant = 53.352 ft lbf/lbm °R (287 J/kg K) for air and 85.78 ft lbf/lbm °R (462 J/kg K) for water vapor.

Alternatively, the ideal gas law can also be written as

$$pv = \left(\frac{R^*}{MW}\right)T$$
(2.2.2.b)

where MW is the molecular weight of the substance, and  $R^*$  is the universal gas constant = 1545.32 ft lbf/lbmol °R (8.3144 kJ/kg-mol K), and whose value is independent of the substance.

- I. *Liquid-vapor properties*. Understanding the behavior of substances such as steam or refrigerants during their transition between liquid and vapor phases is very important in air conditioning and refrigeration systems. The ideal gas law cannot be used for such cases and one must use property tables or charts specific to the substance. Properties of liquid and saturated water vapor are given in Table 2.2.2 based on temperature and pressure. The second column gives the corresponding saturation pressure. Specific volume and specific enthalpy at the saturated liquid condition and at the saturated vapor condition are listed in the columns. Separate tables are also available to determine properties of superheated vapor.
- J. A thermodynamic *process* is one where the state of a system under study undergoes a change. For example, the dehumidification of air in a cooling coil is a thermodynamic process. A process is described in part by the series of states passed through by the system. Often some interaction between the system and the surroundings occurs during the process. Thermodynamic analysis basically involves studying the interaction of work, heat, and the properties of the substance contained in the system.

		Tempera Specific v	ture Table vol, cu ft/lb	Ent	halpy, B	tu/lb			Pressure Specific vo	e Table I, cu ft/lb	Ent	halpy, B	tu/lb
		Sat	Sat	Sat		Sat			Sat	Sat	Sat		Sat
Temp	Abs press,	liquid	vapor	liquid	Evap	vapor	Abs press,		liquid	vapor	liquid	Evap	vapor
F	psi	$v_f$	$v_{g}$	$h_{f}$	$h_{fg}$	$h_{g}$	psi	Temp, F	$v_f$	$v_{g}$	$h_{f}$	$h_{fg}$	$h_{g}$
32	0.08854	0.01602	33306	0.00	1075.8	1075.8	0.5	79 58	0.01608	641.4	47.6	1048.8	1096.4
35	0.09995	0.01602	2947	3.02	1074.1	1077.1	1.0	101.74	0.01614	333.6	69.7	1036.3	1106.0
40	0.12170	0.01602	2444	8.05	1071.3	1079.3	2.0	126.08	0.01623	173.73	94.0	1022.2	1116.2
45	0.14752	0.01602	2036.4	13.06	1068.4	1081.5	3.0	141.48	0.01630	118.71	109.4	1013.2	1122.6
50	0.17811	0.01603	1703.2	18.07	1065.6	1083.7	4.0	152.97	0.01636	90.63	120.9	1006.4	1127.3
55	0.2141	0.01603	1430.7	23.07	1062.7	1085.8	5.0	162.24	0.01640	73.53	130.1	1001.0	1131.1
60	0.2563	0.01604	1206.7	28.06	1059.9	1088.0	6.0	170.06	0.01645	61.98	138.0	996.2	1134.2
65	0.3056	0.01605	1021.4	33.05	1057.1	1090.2	7.0	176.85	0.01649	53.64	144.8	992.1	1136.9
70	0.3631	0.01606	867.9	38.04	1054.3	1092.3	8.0	182.86	0.01653	47.34	150.8	988.5	1139.3
75	0.4298	0.01607	740.0	43.03	1051.5	1094.5	9.0	188.28	0.01656	42.40	156.2	985.2	1141.4
80	0.5069	0.01608	633.1	48.02	1048.6	1096.6	10	193.21	0.01659	38.42	161.2	982.1	1143.3
85	0.5959	0.01609	543.5	53.00	1045.8	1098.8	14.7	212.00	0.01672	26.80	180.0	970.4	1150.4
90	0.6982	0.01610	468.0	57.99	1042.9	1100.9	20	227.96	0.01683	20.089	196.2	960.1	1156.3
95	0.8153	0.01612	404.3	62.98	1040.1	1103.1	25	240.07	0.01692	16.303	208.5	952.1	1160.6
100	0.9492	0.01613	350.4	67.97	1037.2	1105.2	30	250.33	0.01701	13.746	218.8	945.3	1164.1
105	1 1016	0.01615	304 5	72.95	1034 3	1107.3	40	267.25	0.01715	10.498	236.0	933 7	1169.7
110	1.1010	0.01617	265.4	77.94	1031.6	1107.5	50	281.01	0.01713	8 515	250.0	924.0	1174.1
115	1.2740	0.01618	231.9	82.93	1028.7	111116	60	201.01	0.01727	7 175	262.1	915.5	1177.6
120	1.4705	0.01620	203.27	87.92	1025.8	11137	70	302.92	0.01748	6 206	272.6	907.9	1180.6
125	1.0524	0.01622	178.61	92.91	1022.0	1115.7	80	312.03	0.01740	5 472	282.0	901.1	1183.1
120	119 120	0.01022	170101	/2//1	102212	111010	00	012100	0101707	011/2	20210	20111	110011
130	2.2225	0.01625	157.34	97.9	1020.0	1117.9	90	320.27	0.01766	4.896	290.6	894.7	1185.3
135	2.5370	0.01627	138.95	102.9	1017.0	1119.9	100	327.81	0.01774	4.432	298.4	888.8	1187.2
140	2.8886	0.01629	123.01	107.9	1014.1	1122.0	110	334.77	0.01782	4.049	305.7	883.2	1188.9
145	3.281	0.01632	109.15	112.9	1011.2	1124.1	120	341.25	0.01789	3.728	312.4	877.9	1190.4
150	3.718	0.01634	97.07	117.9	1008.2	1126.1	130	347.32	0.01796	3.455	318.8	872.9	1191.7
155	4.203	0.01637	86.52	122.9	1005.2	1128.1	140	353.02	0.01802	3.220	324.8	868.2	1193.0
160	4.741	0.01639	77.29	127.9	1002.3	1130.2	150	358.42	0.01809	3.015	330.5	863.6	1194.1
165	5.335	0.01642	69.19	132.9	999.3	1132.2	160	363.53	0.01815	2.834	335.9	859.2	1195.1
170	5.992	0.01645	62.06	137.9	996.3	1134.2	170	368.41	0.01822	2.675	341.1	854.9	1196.0
175	6./15	0.01648	55.78	142.9	993.3	1136.2	180	3/3.06	0.01827	2.532	346.1	850.8	1196.9
180	7.510	0.01651	50.23	147.9	990.2	1138.1	190	377.51	0.01833	2.404	350.8	846.8	1197.6
185	8.383	0.01654	45.31	152.9	987.2	1140.1	200	381.79	0.01839	2.288	355.4	843.0	1198.4
190	9.339	0.01657	40.96	157.9	984.1	1142.0	250	400.95	0.01865	1.8438	376.0	825.1	1201.1
200	11.526	0.01663	33.64	168.0	977.9	1145.9	300	417.33	0.01890	1.5433	393.8	809.0	1202.8
212	14.696	0.01672	26.80	180.0	970.4	1150.4	350	431.72	0.01913	1.3260	409.7	794.2	1203.9
220	17 186	0.01677	23 15	188 1	965.2	1153.4	400	444 59	0.0193	1 1613	424.0	780 5	1204 5
240	24 969	0.01692	16 323	208.3	952.2	1155.4	450	456.28	0.0195	1.1015	437.2	767.4	1204.5
260	35 429	0.01709	11 763	200.5	938.7	1167.3	500	467.01	0.0197	0.9278	449.4	755.0	1201.0
280	49.203	0.01726	8.645	249.1	924.7	1173.8	600	486.21	0.0201	0.7698	471.6	731.6	1201.1
300	67.013	0.01745	6.466	269.6	910.1	1179.7	700	503.10	0.0205	0.6554	491.5	709.7	1201.2
	-						-						. –
350	134.63	0.01799	3.342	321.6	870.7	1192.3	800	518.23	0.0209	0.5687	509.7	688.9	1198.6
400	247.31	0.01864	1.8633	375.0	826.0	1201.0	900	531.98	0.0212	0.5006	526.6	668.8	1195.4
450	422.6	0.0194	1.0993	430.1	774.5	1204.6	1000	544.61	0.0216	0.4456	542.4	649.4	1191.8
500	680.8	0.0204	0.6749	487.8	713.9	1201.7	1200	567.22	0.0223	0.3619	571.7	611.7	1183.4
550	1045.2	0.0218	0.4240	549.3	640.8	1190.0	1500	596.23	0.0235	0.2760	611.6	556.3	1167.9

 TABLE 2.2.2
 Properties of Saturated Steam and Saturated Water



FIGURE 2.2.1 A simple flow system.



FIGURE 2.2.2 Schematic thermodynamic system for steady flow.

- K. *Equilibrium* is a condition of balance maintained by an equality of opposing forces. There are different types of equilibrium: thermal, mechanical, chemical. The study of thermodynamics has to do with determining end states, and not with the dynamics of the process (e.g., how fast the process change occurs). The concept of equilibrium is important as it is only in an equilibrium state that the thermodynamic properties have meaning. We implicitly assume that the system moves from one state of equilibrium to another very slowly as it undergoes a process, a condition called quasi-equilibrium or a quasi-static process.
- L. *Energy* is the capacity for producing an effect. It can be stored within the system as potential energy, kinetic energy, internal energy, etc. It can also be transferred to or from the system by work or heat transfer. Heat is transferred across the boundary of a system to another system or surroundings by virtue of a temperature difference between the two systems.
- M. Conservation of mass simply states that mass of a substance can be neither created nor destroyed in the processes analyzed. Consider a simple flow system (shown in Figure 2.2.1) when a fluid stream flows into and out of a control volume. If the mass in the system at time  $\theta$  is m( $\theta$ ), then the mass at time  $\theta + \delta\theta$  is m( $\theta + \delta\theta$ ). Assuming that during the time increment d $\theta$ , an increment  $\delta m_{in}$  enters the system and  $\delta m_{out}$  leaves the system, conservation of mass relation results in

$$m(\theta) + \delta m_{in} = m(\theta + \delta \theta) + \delta m_{out}$$

which can be simplified as

$$\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}\theta} = \dot{\mathbf{m}}_{\mathrm{in}} - \dot{\mathbf{m}}_{\mathrm{out}} \tag{2.2.3}$$

where  $\dot{m} = \frac{\delta m}{\delta \theta}$ 

For steady flow, 
$$\frac{dm}{d\theta} = 0$$
 and  $\dot{m}_{in} = \dot{m}_{out}$  (2.2.4)

N. *Conservation of energy* or the first law of thermodynamics is the basis of most of the analysis done in HVAC. It formulates a relationship between the various quantities shown in Figure 2.2.2. It is described in Chapter 2.1.

# 2.2.3 Psychrometric Properties of Moist Air

Psychrometry is the study of the properties of moist air, i.e., a mixture of air and water vapor. A thorough understanding of psychrometry is essential since it is fundamental to understanding the various processes related to air conditioning. Atmospheric air is never totally dry; it always contains varying degrees of water vapor. Just like relatively small amounts of trace materials drastically impact the physical properties of steel alloys, small amounts of moisture have a large influence on human comfort.

The amount of water vapor contained in air may vary from near zero (totally dry) to a maximum determined by the temperature and pressure of the mixture. Properties of moist air can be determined from tables such as Table 2.2.3, from equations and steam tables as shown below, or from the psychrometric chart (described later). Moist air up to about three atmospheres pressure can be assumed to obey the perfect gas law. Assuming dry air to consist of one gas only, the total pressure  $p_t$  of moist air, given by the Gibbs-Dalton Law for a mixture of perfect gases, is equal to the individual contributions of dry air and water vapor.

$$p_t = p_a + p_v \tag{2.2.5}$$

where  $p_a$  is the partial pressure of dry air, and  $p_v$  is the partial pressure of water vapor. It is because  $p_v \ll p_a$  that we can implicitly assume water vapor also follows the perfect gas law for atmospheric air.

The thermodynamic state of an air-vapor mixture is fully determined if three independent intensive properties are specified. Since one can assume for most of the HVAC processes being studied that the total atmospheric pressure does not change, a chart known as the psychrometric chart, applicable to a specific value of total pressure (commonly the standard atmospheric pressure), is used. The psychrometric chart not only provides a quick means for determining values of moist air properties, it is also very useful in solving numerous process problems with moist air and allows quick visualization of how the process occurs. Hence, for better comprehension, we describe the manner in which it is generated along with the description of the pertinent moist air properties.

The primary moist air properties shown on a psychrometric chart are described below:

- A. Dry-bulb temperature  $T_{db}$  or t is the temperature of air one would measure with an ordinary thermometer. This property is the x-axis of the psychrometric chart (Figure 2.2.3).
- B. Saturation pressure of water vapor:  $p_s$  or  $p_{v,sat}$  can be determined or obtained from steam tables (see Table 2.2.2). For example, the saturation pressure  $p_s$  of water vapor from Table 2.2.2 at a temperature  $t = 80^{\circ}F$  is  $p_s = 0.5069$  psia, which is about 30 times less than the corresponding partial pressure of dry air  $p_a$ .
- C. *Humidity ratio* or specific humidity or absolute humidity W is defined as the ratio of the mass of water vapor to that of dry air, i.e.,

$$W = \frac{\text{mass of water vapor}}{\text{mass of dry air}}$$

Using the ideal gas law under saturated air conditions,

$$W = \frac{m_v}{m_a} = \frac{p_s VMW_v / R^T}{p_a VMW_a / R^*T} = \frac{p_s MW_v}{(p_t - p_s)MW_a}$$

where V is an arbitrary volume of the air and water vapor mixture,  $R^*$  is the universal gas constant,  $MW_a$  is the molecular weight of dry air (28.965), and  $MW_v$  is the molecular weight of water (18.015). The above formula then reduces to

$$W = 0.622 \frac{p_s}{p_t - p_s}$$
(2.2.6)

											Condense	ed Water		
	Humidity								_			Entropy	Vapor	
	Ratio	C	Volume			Enthalpy		р. (	Entropy	T)	Enthalpy	Btu/	Press.	
Temp.	$lb_w/lb_a$	n	3/Ib dry a	ır		But/Ib dry	air	But	t/(Ib dry ai	r·F)	But/lb	(lb·F)	in Hg	Temp.
F	$W_s$	$v_a$	$v_{as}$	$\nu_s$	$h_a$	$h_{as}$	$h_s$	s <sub>a</sub>	\$ <sub>as</sub>	<i>S</i> <sub>5</sub>	$h_w$	$S_w$	$p_s$	F
0	0.0007875	11.579	0.015	11.594	0.0	0.835	0.835	0.00000	0.00192	0.00192	-158.89	-0.3243	0.037671	0
5	0.0010207	11.706	0.019	11.725	1.201	1.085	2.286	0.00260	0.00247	0.00506	-156.52	-0.3192	0.048814	5
10	0.0013158	11.832	0.025	11.857	2.402	1.402	3.804	0.00517	0.00315	0.00832	-154.13	-0.3141	0.062901	10
15	0.0016874	11.959	0.032	11.991	3.603	1.801	5.404	0.00771	0.00400	0.01171	-151.71	-0.3089	0.080623	15
20	0.0021531	12.085	0.042	12.127	4.804	2.303	7.107	0.01023	0.00505	0.01528	-149.27	-0.3038	0.102798	20
25	0.0027339	12.212	0.054	12.265	6.005	2.930	8.935	0.01272	0.00636	0.01908	-146.80	-0.2987	0.130413	25
30	0.0034552	12.338	0.068	12.406	7.206	3.711	10.917	0.01519	0.00796	0.02315	-144.31	-0.2936	0.164631	30
32	0.0037895	12.389	0.075	12.464	7.687	4.073	11.760	0.01617	0.00870	0.02487	-143.30	-0.2915	0.180479	32
36	0.004452	12.490	0.089	12.579	8.648	4.793	13.441	0.01811	0.01016	0.02827	4.05	0.0081	0.21181	36
40	0.005216	12.591	0.105	12.696	9.609	5.624	15.233	0.02004	0.01183	0.03187	8.07	0.0162	0.24784	40
44	0.006094	12.692	0.124	12.816	10.570	6.582	17.152	0.02196	0.01374	0.03570	12.09	0.0242	0.28918	44
48	0.007103	12.793	0.146	12.939	11.531	7.684	19.215	0.02386	0.01592	0.03978	16.10	0.0321	0.33651	48
52	0.008259	12.894	0.171	13.065	12.492	8.949	21.441	0.02575	0.01840	0.04415	20.11	0.0400	0.39054	52
56	0.009580	12.995	0.200	13.195	13.453	10.397	23.850	0.02762	0.02122	0.04884	24.11	0.0478	0.45205	56
60	0.011087	13.096	0.233	13.329	14.415	12.052	26.467	0.02947	0.02442	0.05389	28.11	0.0555	0.52193	60
64	0.012805	13.198	0.271	13.468	15.376	13.942	29.318	0.03132	0.02804	0.05936	32.11	0.0632	0.60113	64
68	0.014758	13.299	0.315	13.613	16.337	16.094	32.431	0.03315	0.03214	0.06529	36.11	0.0708	0.69065	68
72	0.016976	13.400	0.365	13.764	17.299	18.543	35.841	0.03496	0.03677	0.07173	40.11	0.0783	0.79167	72
76	0.019491	13.501	0.422	13.923	18.260	21.323	39.583	0.03676	0.04199	0.07875	44.10	0.0858	0.90533	76
80	0.022340	13.602	0.487	14.089	19.222	24.479	43.701	0.03855	0.04787	0.08642	48.10	0.0933	1.03302	80
84	0.025563	13.703	0.561	14.264	20.183	28.055	48.238	0.04033	0.05448	0.09481	52.09	0.1006	1.17608	84
88	0.029208	13.804	0.646	14.450	21.145	32.105	53.250	0.04209	0.06192	0.10401	56.09	0.1080	1.33613	88
92	0.033323	13.905	0.742	14.647	22.107	36.687	58.794	0.04384	0.07028	0.11412	60.08	0.1152	1.51471	92
96	0.037972	14.006	0.852	14.858	23.069	41.871	64.940	0.04558	0.07968	0.12525	64.07	0.1224	1.71372	96
100	0.043219	14.107	0.976	15.084	24.031	47.730	71.761	0.04730	0.09022	0.13752	68.07	0.1296	1.93492	100
104	0.049140	14.208	1.118	15.326	24.993	54.354	79.346	0.04901	0.10206	0.15108	72.06	0.1367	2.18037	104
108	0.055826	14.309	1.279	15.588	25.955	61.844	87.799	0.05071	0.11537	0.16608	76.05	0.1438	2.45232	108
112	0.063378	14.411	1.462	5.872	26.917	70.319	97.237	0.05240	0.13032	0.18272	80.05	0.1508	2.75310	112
116	0.071908	14.512	1.670	16.181	27.879	79.906	107.786	0.05408	0.14713	0.20121	84.04	0.1577	3.08488	116
120	0.081560	14.613	1.906	16.519	28.842	90.770	119.612	0.05575	0.16605	0.22180	88.04	0.1647	3.45052	120
124	0.092500	14.714	2.176	16.890	29.805	103.102	132.907	0.05740	0.18739	0.24480	92.03	0.1715	3.85298	124
128	0.104910	14.815	2.485	17.299	30.767	11/.111	14/.8/8	0.05905	0.21149	0.2/054	96.03	0.1/83	4.294//	128
132	0.119023	14.916	2.837	17.753	31./30	133.066	164./96	0.06068	0.238/6	0.29944	100.02	0.1851	4.//919	132
136	0.135124	15.01/	3.242	18.259	32.693	151.294	183.98/	0.06230	0.269/3	0.33203	104.02	0.1919	5.309/3	136
140	0.155558	15.118	5.708	18.825	24.620	1/2.168	205.824	0.06591	0.30498	0.36890	108.02	0.1985	5.88945	140
144	0.1/4694	15.219	4.245	19.464	34.620	196.185	250.802	0.06551	0.34550	0.41081	112.02	0.2052	0.52241	144
148	0.199110	15.520	4.869	20.189	26 5 46	223.932	259.514	0.06/10	0.39160	0.458/1	116.02	0.2118	7.21239	148
152	0.22/429	15.421	5.590	21.017	27 5 10	202.040	292.705	0.00000	0.44507	0.515/5	120.02	0.2184	7.90500	152
150	0.200312	15.522	0.450 7.450	21.972	20 474	295.849	276 727	0.07025	0.50725	0.57749	124.02	0.2249	0.77915	150
160	0.29945	15.625	7.459 8.664	23.082	20.429	201.005	3/0./3/	0.07181	0.58007	0.05188	128.02	0.2314	9.6648	160
164	0.34372	15.724	0.004	24.300	10 402	454 620	450.555	0.07337	0.00022	0.73939	132.03	0.2376	10.0230	164
100	0.40131	15.025	11.004	23.942	40.402	522 120	495.052 573 504	0.07491	0.70923	0.04413	130.03	0.2442	12 7880	100
172	0.40903	15.920	11.094	27.020	41.300	628 107	670 528	0.07044	1.04828	1 1 2624	140.04	0.2560	12.7000	172
180	0.55274	16 128	16 000	33 037	42.331	7/0 271	793 166	0.07790	1.04020	1.12024	144.05	0.2309	15 3007	180
184	0.03911	16 220	20 564	36 702	43.273	742.071 908.061	952 221	0.0794/	1.24230	1.52105	152.00	0.2032	16 7100	100
188	0.79703	16 330	20.004	41 878	44.200	1121 174	1166 300	0.08098	1.49002	1,97450	156.09	0.2094	18 2357	189
100	1 24471	16 / 31	23.490	48 909	46 190	1422 047	1468 238	0.08306	2 30103	2 38580	160.00	0.2750	19 8652	100
194	1.64070	16 532	13 016	59 578	47 155	1877 032	1924 188	0.085/3	3 01244	3 00707	164.12	0.2010	21.6152	192
200	2.30454	16.633	60.793	77.426	48.121	2640.084	2688.205	0.08690	4.19787	4.28477	168.13	0.2941	23.4906	200
-00		-0.000	00.799		10.141		2000.200	0.00000			100.10	J		-00

TABLE 2.2.3 Moist Air, Standard Atmospheric Pressure, 14.696 psi

Source: Abridged by permission from ASHRAE Handbook, Fundamentals Volume, 1985.



**FIGURE 2.2.3** Construction of psychrometric chart, showing lines of constant property values. (*a*) Lines of constant dry-bulb temperature (DB) on the psychrometric chart. (*b*) Lines of constant humidity ratio (W) on the psychrometric chart. (*c*) Lines of constant relative humidity (RH) on the psychrometric chart. (*d*) Lines of constant specific volume (v) on the psychrometric chart. (*e*) Lines of constant wet-bulb temperature (WB) on the psychrometric chart. (*f*) Lines of constant enthalpy (*h*) on the psychrometric chart.

Note that because  $p_s \ll p_t$ , the relation between W and  $p_s$  is close to being linear in  $p_s$ . As shown in Figure 2.2.3, the y-axis of the psychrometric chart is allocated to the humidity ratio.

D. *Relative humidity* RH (or  $\phi$ ) is defined as the ratio of the partial pressure of water vapor  $p_v$  divided by the saturated pressure of water vapor at the same dry-bulb temperature, i.e.,

$$\phi = \frac{p_v}{p_s} \tag{2.2.7}$$

As shown in Figure 2.2.3(c), one can now add lines of constant RH by marking vertical distances between the saturation line and the base of the chart.

E. *Specific enthalpy* h of moist air is equal to the sum of dry air enthalpy and that of water vapor. Though the air-vapor mixture is likely to be superheated, there is not much error in assuming the enthalpy of the water vapor to be equal to the saturated value at the same temperature. Thus, it is convenient to estimate moist air enthalpy as

$$\mathbf{h} = \mathbf{c}_{\mathrm{p}}\mathbf{t} + \mathbf{W} \cdot \mathbf{h}_{\mathrm{g}} \tag{2.2.8a}$$

where  $h_g$  is the enthalpy of saturated steam at temperature t which can be determined from Table 2.2.2. Enthalpy values are always based on a datum value, usually 32°F or 0°C. Alternative expressions for moist air enthalpy are:

SI units: 
$$h = t + W(2501.3 + 1.86 \cdot t)$$
 in kJ/kg of dry air (2.2.8b)  
IP units:  $h = 0.24 \cdot t + W(1061.2 + 0.444 \cdot t)$  in Btu/lb.

As shown in Figure 2.2.3(f), lines of constant enthalpy can now be drawn since h is a function of dry-bulb temperature and specific humidity.

F. *Specific volume* v is the volume of the mixture (say in m<sup>3</sup> or in ft<sup>3</sup>) per unit mass of dry air. The perfect gas law can be used to estimate it:

$$v = \frac{R_a T}{p_t - p_s}$$
(2.2.9)

Lines of constant specific volume are shown in Figure 2.2.3(d) by selecting a value of v and solving for  $p_s$  (and hence W using Equation 2.2.6) for different values of T. Recall that a specific psychrometric chart can be used for only a specific pressure  $p_t$ .

G. Adiabatic saturation temperature is the temperature reached by air when it passes through a spray of water such that there is thermal and vapor pressure equilibrium between the air and the water, as shown in Figure 2.2.4. The process is assumed to be adiabatic in that no heat is either added or lost from the chamber. Since the air is not fully saturated, a certain amount of water evaporates into the air whose latent heat is supplied by the air. Hence the moist air dry-bulb temperature decreases while its specific humidity increases. A small amount of make-up water to compensate for the evaporated water is supplied at a temperature equal to that of the sump.



FIGURE 2.2.4 Adiabatic saturation.



FIGURE 2.2.5 Sling psychrometer device for conveniently measuring wet- and dry-bulb temperatures.

H. The *adiabatic wet-bulb temperature* is the temperature of sump water in an adiabatic saturator. An energy balance on the adiabatic saturator shown in Figure 2.2.4 yields

$$h_1 = h_2 - (W_2 - W_1) \cdot h_f \tag{2.2.10}$$

where  $h_f$  is the enthalpy of saturated liquid at the thermodynamic wet-bulb temperature. Because of the latent heat term, lines of constant wet-bulb temperature are not quite identical to lines of constant enthalpy, though close. Lines of constant wet-bulb temperature are shown in Figure 2.2.3(e).

The adiabatic saturator is not a practical device for measuring the adiabatic saturation temperature. Instead, the hand-held sling psychrometer is often used (Figure 2.2.5). The apparatus consists of two thermometers, one measuring the dry-bulb temperature, and the other, which has a wetted wick covering the bulb, measures the wet-bulb temperature. The instrument has a handle which allows the thermometers to be spun so as to induce air movement over the bulbs that is adequate for proper heat transfer between the bulb and the ambient air. Though the wet-bulb temperature is not the same as the adiabatic saturation temperature, the difference is small. A detailed discussion of these differences can be found in Kuehn et al. (1998). Electronic devices are also commonly used to measure humidity levels in air.

I. Dew point temperature  $T_{dp}$  of a given mixture is the temperature of *saturated* moist air at the same pressure, temperature, and humidity ratio as the given mixture. When a surface reaches the dew point temperature, the moisture will start condensing from the surrounding air.

Note that specific volume, specific enthalpy, and specific humidity are all defined per mass of *dry* air and not per mass of moist air. This convention dispels the confusion created when mass transfer takes place (i.e., when water vapor is either added to or removed from the air).

A complete psychrometric chart corresponding to standard atmospheric pressure is shown in Figure 2.2.6. Although computer programs allow for more accurate and faster determination of the moist air properties (as well as the conversion of different systems of units and arbitrary atmospheric pressures), the psychrometric chart is still used extensively by HVAC professionals for several aspects of design and analysis.



FIGURE 2.2.6 ASHRAE Psychrometric Chart No. 1. With permission.

Example 1: Determination of moist air properties.

Calculate values of humidity ratio, specific volume, and enthalpy for air at  $60^{\circ}$ F and RH = 80% using the equations presented above. Assume standard atmospheric pressure.

From the steam tables, saturated vapor pressure at 60°F:  $p_s = 0.2563$  psia The vapor pressure of the moist air:  $p_v = p_s \times RH = 0.2563 \times 0.8 = 0.2050$  psia

Humidity ratio: W =  $0.622 \frac{p_v}{P - p_v} = 0.622 \frac{0.205}{14.696 - 0.205} = 0.0088$  lbw/lba

Density:  $\frac{1}{v} = \rho = \frac{p_a}{R_a T} = \frac{(14.696 - 0.205)}{53.352 \cdot (60 + 459.67)} \times 144 = 0.07526 \text{ lba/ft}^3$ 

(the multiplier 144 converts  $ft^2$  into  $in^2$ ). Thus specific volume:  $v = (1/0.07526) = 13.287 \text{ ft}^3/\text{lba}$ . Finally, enthalpy is determined from

$$h = 0.240 \cdot t + W (1061.2 + 0.444 t)$$

 $= 0.240 \times 60 + 0.0088 (1061.2 + 0.444 \times 60) = 110.13$  Btu/lba.

#### 2.2.4 Psychrometric Processes

Analysis of moist air processes with various HVAC devices essentially involves a few fundamental processes, discussed below. Consider a duct containing a device through which moist air is flowing. The device could be a cooling or heating coil and/or a humidifier. The analysis of moist air processes flowing through such a device is based on the laws of conservation of mass and energy. Although in actual practice the properties of the moist air may not be uniform across the duct cross section (especially downstream of the device), such phenomena are neglected, and the focus is on bulk or fully mixed conditions. Further, assuming (1) steady state conditions and (2) a perfectly insulated duct, the following equations apply:

Mass balance of dry air:	$m_{a1} = m_{a2}$	(2.2.11a)
Mass balance on water vapor:	$m_{a1}W_1 + m_w = m_{a2}W_2$	(2.2.11b)
Heat balance:	$m_{a1}h_1 + m_w h_w + Q = m_{a2}h_2$	(2.2.11c)

where Q is the rate of heat added to the air stream (in W or Btu/hr),

 $m_{\rm a}$  is the mass flow rate of dry air (in kga/s or lba/hr), and

m<sub>w</sub> is the rate of water added to the air stream.

## A. Sensible Heating and Cooling

A process is called *sensible* (either heating or cooling) when it involves a change in dry-bulb temperature only (i.e., the moisture content specified by the specific humidity is unchanged in a sensible heating or cooling process). This could apply to either heating (an increase in  $T_{db}$ ) or to cooling (a decrease in  $T_{db}$ ). In such a case,  $m_w = 0$ , and  $W_1 = W_2$ . The above equations reduce to

$$Q = m_a (h_2 - h_1) 2.2.12$$

where  $m_a = m_{a1} = m_{a2}$ 



DRY-BULB TEMPERATURE



The process of sensible heating or cooling is represented as a straight line on the psychrometric chart as shown in Figure 2.2.7. Such a process occurs when moist air flows across a cooling coil when condensation does not occur.

#### Example 2: Sensible heating

Moist air enters a steam-heating coil at 40°F dry-bulb temperature and 36°F wet-bulb temperature at a rate of 2000 ft<sup>3</sup>/min. The air leaves the coil at a dry-bulb temperature of 140°F. Determine the heat transfer rate that occurs at the coil.

Assumptions: steady state, standard atmospheric conditions

Heat transfer rate  $Q = m_a (h_2 - h_1)$ 

where states 1 and 2 represent the entering and exiting air stream conditions, respectively. From the psychrometric chart, we find:  $v_1 = 12.66$  ft<sup>3</sup>/lba,  $h_1 = 13.47$  Btu/lba,  $h_2 = 37.70$  Btu/lba

The mass flow rate of moist air:  $m_a = \frac{Q_a}{v_1} = \frac{2000}{12.66} = 158$  lba/min = 9479 lba/hr

Finally,  $Q = 9479 \cdot (37.70 - 13.47) = 229,670$  Btu/hr.

### **B.** Cooling and Dehumidification

This process occurs when conditioning outdoor air in summer or in internal spaces where heat and moisture are removed by cooling coils in a conditioned space. For this process to occur, moist air is cooled to a temperature below its dew point. Some of the water vapor condenses out of the air stream. Although the actual process path varies depending on the type of surface, surface temperature, and flow conditions, the heat and mass transfer can be expressed in terms of the initial and final states.

As shown in Figure 2.2.8, a certain amount of moisture condenses out of the air stream. Although this condensation occurs at various temperatures ranging from the initial dew point to its final saturation temperature, it is assumed that condensed water is cooled to the final air temperature  $t_2$  before it drains out. The above equations reduce to

Rate of water condensation:	$m_w = m_a(W_1 - W_2)$	(2.2.13a)
Rate of total heat transfer:	$Q = m_a[(h_1 - h_2) - (W_1 - W_2)h_{w^2}]$	(2.2.13b)







The above equation gives the total rate of heat transfer from the moist air. The last term is usually small compared to the other term and is often neglected (as illustrated in Example 3). Cooling and dehumidification processes involve both sensible and latent heat transfer. The sensible heat transfer  $q_s$  results in a decrease in dry-bulb temperature, while the latent heat transfer  $q_l$  is associated with the decrease in specific humidity. These quantities can be estimated as follows. Let point "a" be the intersection point between the constant dry-bulb temperature line from point 1 and the constant specific humidity line at point 2 (see Figure 2.2.8). Then

Rate of sensible heat transfer: 
$$Q_s = m_a(h_a - h_2)$$
 (2.2.14a)

Rate of latent heat transfer:  $Q_1 = m_a(h_1 - h_a)$  (2.2.14b)

It is customary to characterize the relative contributions of sensible and latent heat transfer rates by the sensible heat ratio (SHR) where SHR is defined as follows:

$$SHR = \frac{q_s}{q_s + q_l}$$
(2.2.15)

Example 3: Cooling and dehumidification.

Moist air enters a cooling coil at 80°F dry-bulb temperature and 67°F wet-bulb. It is cooled to 58°F and 80% RH. The volume flow rate is 2000 cfm and the condensate leaves at 60°F. Find the total coil heat transfer rate, as well as the individual sensible and latent heat transfer rates.

Assumptions: steady state, standard atmospheric conditions

Heat transfer rate

$$Q = m_a(h_2 - h_1) + m_w \cdot h_w$$

Note that Q will be negative, since cooling is being done. From the psychrometric chart:  $\begin{array}{l} h_1 &= 31.6 \mbox{ Btu/lba,} \\ h_2 &= 22.9 \mbox{ Btu/lba,} \\ v_1 &= 13.85 \mbox{ ft}^3/lba, \\ W_1 &= 0.0112 \mbox{ lbw/lba,} \\ W_2 &= 0.0082 \mbox{ lbw/lba,} \end{array}$ 

and enthalpy of liquid water at 60°F from the steam tables (Table 2.2.2),

The mass flow rate of dry air:

$$m_a = \frac{Q_a}{v_1} = \frac{2000}{13.85} = 144.4 \text{ lba/min} = 8664 \text{ lba/hr}$$

Rate at which water is condensed:

 $m_w = m_a \cdot (W_1 - W_2) = 8664 \cdot (0.0112 - 0.0082) = 25.99 \text{ lbw/hr}$ 

Finally,

$$Q = 8664 \cdot (31.6 - 22.9) - 25.99 \times 28.06 = 75379.06 - 729.3 = 74,650 \text{ Btu/hr}$$
$$Q = 6.22 \text{ tons}$$

(As stated in the text above, the contribution of the heat contained in the condensing water is very small; here it is about 1% only). Often this term is ignored in psychrometric calculations.

In order to determine the contributions of the sensible and latent heat transfer rates, one determines point "a" on the psychrometric chart. This can be done by drawing a horizontal line from point 2 (constant humidity ratio) and a vertical line from point 1 (constant dry-bulb temperature). The enthalpy at this point  $h_a = 27$  Btu/lba.

Sensible heat transfer rate:

$$q_s = m_a \cdot (h_a - h_2) = 8664 \cdot (27 - 22.9) = 35,522 \text{ Btu/hr} = 2.96 \text{ tons}$$

Latent heat transfer rate:

$$q_1 = m_a \cdot (h_1 - h_a) = 8664 \cdot (31.6 - 27) = 39,850 \text{ Btu/hr} = 3.32 \text{ tons}$$

The sensible heat ratio is

$$SHR = 35,522 / (35,522 + 39,854) = 0.47$$

## C. Heating and Humidification

In winter when the outdoor air is cool and dry, the building air supply stream must be heated and humidified to meet comfort criteria. In this case, we have

Rate of water evaporation:	$m_w = m_a (W_2 - W_1)$	(2.2.16a)
Rate of heat transfer:	$Q = m_a[(h_2 - h_1) - (W_2 - W_1)h_{w2}]$	(2.2.16b)

Alternatively the direct addition of moisture to an air stream without any heat addition can be adopted to humidify an air stream (i.e., there is no heating coil). The heat input term Q is zero in Equation 2.2.16b.

The water is typically in the form of spray or vapor (steam). Practical examples of this process include humidifiers, cooling towers, and evaporative coolers.

The direction of the conditioning line between states 1 and 2 depends on the enthalpy of the moisture added. Two unique cases will be discussed. The cases depend on the state of the humidifying spray. First, if the spray is saturated vapor ( $h_w = h_g$ ) at the same dry-bulb temperature of the incoming air stream, the process line will proceed at a constant dry-bulb temperature (line A of Figure 2.2.7). The enthalpy and wet-bulb temperature of the air will increase, but the dry-bulb temperature of the air will remain constant.

Another case occurs if the humidifying spray is saturated water at the wet-bulb temperature of the air. The exiting air stream will have the same wet-bulb temperature as the entering air stream. The dry-bulb temperature of the air will decrease. The leaving enthalpy of the air will be close to the entering enthalpy because the constant enthalpy lines on the psychrometric chart are approximately parallel to the constant wet-bulb lines. This process is shown by line H in Figure 2.2.7.

If the enthalpy of the spray is larger than the enthalpy of saturated water vapor at the dry-bulb temperature, then the air stream will be sensibly heated during humidification. In contrast, if the spray enthalpy is less than the enthalpy of saturated water vapor at the entering dry-bulb temperature of the air, the air will be sensibly cooled during the humidification process.

Example 4: Heating and humidification.

Outdoor winter air enters a heating and humidification system at a rate of 900 cfm and at 32°F dry-bulb and 28°F wet-bulb conditions. The air absorbs 75,000 Btu/hr of energy and 25 lb/hr of saturated steam at 212°F. Determine the dry-bulb and wet-bulb temperature of the leaving air.

Determine relevant properties:

$$h_1 = 10.2 \text{ Btu/lb}, v_1 = 12.42 \text{ ft}^3/\text{lba}, W_1 = 0.0023 \text{ lbw/lba}$$

The mass flow rate of dry air:

$$m_a = \frac{Q}{v_1} = \frac{900}{12.42} = 72.45$$
 lba/min

Equation 2.2.16a can be used to solve for the final humidity ratio:

$$W_2 = \frac{m_a W_1 + m_w}{m_a} = W_1 + \frac{m_w}{m_a} = 0.0023 + \frac{25}{72.45 \times 60} = 0.008051 \text{ lbw/lba}$$

Equation 2.2.16b is used to determine the exit air enthalpy:

$$h_2 = h_1 + \frac{q}{m_a} + \frac{m_w}{m_a} h_w = 10.2 + \frac{75000}{72.45 \times 60} + \frac{25}{72.45 \times 60} \cdot 1150.5 = 34.07 \text{ Btu/lba}$$

With the exit humidity ratio known, the final state is established and other properties can be read from the psychrometric chart.

### D. Adiabatic Mixing of Air Streams

The mixing of two air streams is very common in HVAC systems. For example, we assume adiabatic conditions (i.e., no heat transfer across the duct walls). Then we have

Mass balance of dry air:	$m_{a1} + m_{a2} = m_{a3}$	(2.2.17a)
Mass balance on water vapor:	$m_{a1}W_1 + m_{a2}W_2 = m_{a3}W_3$	(2.2.17b)
Heat balance:	$m_{a1}h_1 + m_{a2}h_2 = m_{a3}h_3$	(2.2.17c)

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FIGURE 2.2.9 Schematic adiabatic mixing of two streams of moist air.

Combining the above yields

$$\frac{\mathbf{h}_2 - \mathbf{h}_3}{\mathbf{h}_3 - \mathbf{h}_1} = \frac{\mathbf{W}_2 - \mathbf{W}_3}{\mathbf{W}_3 - \mathbf{W}_1} = \frac{\mathbf{m}_{a1}}{\mathbf{m}_{a2}}$$
(2.2.18)

The state of the mixed streams lies on a straight line between states 1 and 2 with the point 3 determined in inverse proportion to the masses of the two incoming air streams (see Figure 2.2.9):

$$\frac{m_{a1}}{m_{a2}} = \frac{\text{segment } 3 - 2}{\text{segment } 1 - 3}$$

**Example 5**: Adiabatic mixing of air streams.

A stream of 5000 cfm outdoor air at 40°F dry-bulb temperature and 35°F wet-bulb temperature is adiabatically mixed with 15,000 cfm of recirculated air at 75°F dry-bulb and 50% RH. Find the dry-bulb temperature and wet-bulb temperature of the resulting mixture.

The following property data are needed:

$$W_1 = 0.00315$$
 lbw/lba,  $v_1 = 12.65$  ft<sup>3</sup>/lba,  $h_1 = 13.2$  Btu/lba $W_2 = 0.00915$  lbw/lba,  $v_2 = 13.68$  ft<sup>3</sup>/lba,  $h_2 = 28.32$  Btu/lba

Mass flow rate of stream 1:

$$m_{a1} = \frac{Q_1}{v_1} = \frac{500}{12.65} = 395.3 \text{ lba/min}$$

Mass flow rate of stream 2:

$$m_{a2} = \frac{Q_2}{v_2} = \frac{15,000}{13.68} = 1096.5 \text{ lba/min}$$

Humidity ratio of mixed air:

$$W_{3} = \frac{1}{(m_{a1} + m_{a2})}(m_{a1} \cdot W_{1} + m_{a2} \cdot W_{2}) = \frac{395.3 \times 0.00315 + 1096.5 \times 0.00915}{1491.8} = 0.00756 \text{ lbw/lba}$$



FIGURE 2.2.10 Schematic flow processes for an air conditioned space.

The dry-bulb temperature and the enthalpy can be similarly determined. It is left to the reader to verify that:

$$T_{3db} = 66^{\circ}F$$
 and  $h_3 = 24.3$  Btu/lba

## 2.2.5 Psychrometric Analysis of Basic HVAC Systems

Buildings generally consist of a number of rooms which may have different energy and moisture gains or losses — the loads. Loads exhibit both seasonal and diurnal variation. Adjacent rooms with similar loads are usually lumped together into one *zone* which is controlled by one thermostat. Air handlers in an HVAC system can be designed to condition one zone (called single zone systems) or multiple zones (called multiple zone systems). Residences and small commercial buildings are usually designed and operated as single zone spaces.

### Condition Line for a Space

A space is air conditioned to offset the heating and/or cooling loads of the space as a result of envelope heat transmission, ventilation air requirements, and internal loads due to occupants, lights, and equipment. The calculations involved in air conditioning design reduce to the determination of the mass of dry air to be circulated, its dry-bulb temperature, and its humidity level that will result in comfortable indoor conditions for the occupants.

Let  $Q_s$  and  $Q_1$  be the sensible and latent loads on a space to be air conditioned (see Figure 2.2.10). The latent load is due to the sum of all rates of moisture gain designated by  $m_w$ . Assuming steady conditions,

$$Q_s + Q_l = m_a (h_2 - h_1)$$
(2.2.19a)

$$m_w = m_a (W_2 - W_1) \tag{2.2.19b}$$

Thus, the enthalpy-moisture ratio q' is

$$q' = \frac{Q_s + Q_l}{m_w} = \frac{h_2 - h_1}{W_2 - W_1}$$
(2.2.20)

The above equation suggests that for the supply air to satisfy simultaneously both the sensible and latent loads, its condition must lie on a straight line called the *condition line* (or load line). In case the space needs to be cooled, supply temperature  $t_1 < t_2$  and the condition line will appear as shown in Figure 2.2.11. Point 1 must lie on this condition line. How far point 1 is from point 2 is determined by practical considerations, for example, on a prespecified air flow rate or on a prespecified temperature difference between states 1 and 2. During summer conditions, an approximate range of variation of  $(t_1 - t_2)$  is 15–25°F (8–15°C).



FIGURE 2.2.11 The condition line.

As shown in Figure 2.2.11, there is a simple graphical construction for drawing the load line through a specified point on a psychrometric chart. The semicircular protractor in the upper left-hand corner of the chart has a series of marks corresponding to different SHR values (see Figure 2.2.6 and Equation 2.2.15). One determines the SHR for the known design conditions and draws a line through point 2 parallel to the line in the protractor circle corresponding to the calculated SHR.

**Example 6**: The air in a space is to be maintained at comfort conditions of 75°F and 50% RH. The sensible and latent load gains for the space are 89,000 Btu/hr and 52,000 Btu/hr respectively. The supply air to the room is to be at 60°F. Determine the dew point temperature and the required volume flow rate of the supply air.

First, calculate the sensible heat ratio:

$$SHR = \frac{89,000}{89,000 + 52,000} = 0.631$$

Next, locate the return air condition (the same as room conditions) as state 2 on the psychrometric chart. The condition line is drawn from this point with the aid of the chart protractor as a line parallel to the SHR = 0.631 line. Where this line intersects the vertical line representing 60°F dry-bulb temperature is the required supply state 1, i.e., the supply air condition. The dew point temperature is read as 49°F for this condition using Figure 2.2.6.

From the psychrometric chart:  $h_1 = 22.45$  Btu/lba,  $v_1 = 13.25$  ft<sup>3</sup>/lba, and  $h_2 = 28.15$  Btu/lba. A simple heat balance yields the required air mass flow rate:

$$m_a = \frac{89,000 + 52,000}{28.15 - 22.45} = 24,700 \text{ lba/hr}$$

Finally the supply volumetric flow rate is:

$$= \frac{\mathrm{m_a} \cdot \mathrm{v_1}}{60} = \frac{24,700 \times 13.25}{60} = 5,465 \,\mathrm{ft}^3/\mathrm{min}$$

## **Cooling Coil Performance Calculations**

Section 2.2.4B discussed the cooling and dehumidification process. The discussion assumed the inlet and outlet conditions of the air stream to be specified, and the objective was to determine the



FIGURE 2.2.12 Schematic psychrometric chart of a real cooling/dehumidifying process.

associated rates of sensible, latent, and total heat transfer. Here we briefly describe the physical phenomenon that occurs as the air stream flows over the cooling coil, and how this process can be represented on the psychrometric chart. Next, we present a simplified model of characterizing the cooling coil performance.

We need to distinguish between two cases: an ideal process and an actual process. The ideal process corresponds to when all the air molecules of the air stream come in perfect contact with the cooling coil surface and are cooled to the dew point temperature, after which dehumidification occurs along the saturated line to the final state, shown as point *d* on Figure 2.2.12. In the real process, all the air molecules do not come into intimate contact with the cooling coil surface. The cooling coil is basically a heat exchanger, consisting of a series of flat, parallel cooled metal surfaces, which form passages for the air stream to flow through. Often there are several rows of coils in parallel. The temperature of the air flowing through is nonuniform both at a given cross-section and along the depth of the coil. The air particles near the surface of the heat exchanger surface follow the idealized process, while those near the center line of the passage are not fully cooled. This results in dehumidification even though the average air temperature in the passage is above the dew point. Since processes on the psychrometric chart represent average or bulk conditions, the real cooling process as the air stream flows through the cooling coil resembles the curved process line shown in Figure 2.2.12.

The phenomenon described above serves as the basis of a simple means of characterizing the cooling coil performance when subjected to varying inlet conditions. The *bypass factor method* assumes that the passage of the air stream through a cooling coil of given geometric design consists of two streams: one stream  $(m_a')$  that comes into intimate contact with the heat exchanger surface, and another that totally bypasses the cooling coil and hence remains at the condition of the entering air. The former portion of the air stream is cooled along the saturation line to the apparatus dew point temperature  $t_d$  as shown in Figure 2.2.12. The final state of the air leaving the cooling coil can be determined if the relative flow rates of both air streams are known. The coil is characterized by a *coil bypass factor b* defined as

$$b \equiv \frac{m_a - m'_a}{m_a} = \frac{t_2 - t_d}{t_1 - t_d}$$
(2.2.21)

where, as shown in Figure 2.2.12,  $t_1$  and  $t_2$  are the dry-bulb temperatures of the air stream entering and leaving the cooling coil.

The usefulness of this approach lies in the fact that the bypass factor of a given coil is a coil characteristic which remains constant under a wide range of operating temperature conditions; however, it is likely to change with varying air flow rate.



FIGURE 2.2.13 Schematic elementary summer air conditioning system.

#### **Psychrometrics of Single Zone Systems**

This section discusses and illustrates general principles of how the fundamental processes presented in the previous section are applied to buildings. We limit the discussion to three common HVAC systems used to condition one-zone spaces. Treatment of multiple zones and the analysis of more complex HVAC systems can be found in Kreider and Rabl (2001). In the discussion that follows, we assume that the temperature and humidity conditions required for maintaining comfort in the space are specified in advance.

Figure 2.2.13a shows the most elementary HVAC system suitable for summer air conditioning, while Figure 2.2.13b shows the corresponding state points on the psychrometric chart. A certain amount of outdoor air is required for ventilation purposes (to meet indoor comfort criteria). Hence, outdoor air is introduced into the system at (4) while the same amount of air is exhausted at (3). This air is shown as being exhausted directly from the space; it is also common to exhaust it from a convenient location in the ducting downstream of the room. A fan is used to move air through the system. Although only a supply air fan is shown, larger systems might use a supply fan and a return fan. Fans add thermal energy to the air stream due to their inefficiencies and may result in air stream heating of 1–3°F. In simplified psychrometric analyses of HVAC systems, this load is either neglected or combined with that of the conditioned space. Hence states (6) and (1) are shown as identical in Figure 2.2.13b. Filters, which are essential for control of particulate matter carried by the air, do not affect the state of the moist air. A cooling coil is the only processing device; it reduces both the dry-bulb temperature and the humidity of zone supply air.

The simple system shown above has several limitations. First, since the cooling coil is the only processing device, only one property of the moist air can be controlled. In comfort air conditioning systems, this is usually the space dry-bulb temperature. Second, though the system can be designed to meet the peak summer loads, such a system is not suitable for part load operation which occurs most of the year. A reheat coil, as shown in Figure 2.2.14a, is commonly used. Here, the cooling coil reduces the supply air specific humidity level at state (6) to that required at state (1). By doing so, the dry-bulb temperature is lower than that required at state (1) to meet the sensible loads of the space. The heating coil now heats the air stream to state (1) as illustrated in Figure 2.2.14b. Hence, as the outdoor conditions vary over the course of the year, the reheat coil acts as a final regulation device, adding only the required amount of heat to condition the air to the level desired at the supply of the room. However, cooling of the supply air stream and subsequent reheat are wasteful of energy.

One approach to reducing the energy waste is to vary the rate of air supply to the room so the condition of the supply air stream at state (1) is equal to that leaving the cooling coil (state 6). Variable air volume



FIGURE 2.2.14 Schematic summer air conditioning system with reheat.



FIGURE 2.2.15 Schematic winter air conditioning system.

(VAV) systems, in contrast to constant volume (CV) systems, have been widely used for the past 20 years. Another means of reducing energy use is to use a heat recovery device (such as a rotary heat wheel or a run-around coil) between the exhaust air stream and the supply air stream (Kuehn et al., 1998).

During winter operation, heating must be supplied to a space to meet comfort conditions. Outdoor air in winter is much drier than that required for indoor human comfort. Hence, the basic system required for winter air conditioning, shown in Figure 2.2.15a, includes an air washer that allows the specific humidity of the supply air to be controlled. A preheat coil, also shown, prevents water in the air washer from possibly freezing when  $T_5 < 32^{\circ}$ F. The various state points for this system operating in winter are shown on the psychrometric chart (Figure 2.2.15b). The process in the air washer (state points 6 to 7) follows the constant wet-bulb line, shown as t<sup>\*</sup>.

Outdoor conditions and internal loads require that the HVAC system supply heating in winter and cooling in summer. Such an HVAC system would be similar to that shown in Figure 2.2.15a with the added feature that a cooling coil would be placed downstream of the preheat coil. The preheat coil now has the primary responsibility of preventing the mixed air from becoming so cold in winter that the cooling coil is likely to suffer physical damage due to coil frosting.

**Example 7**: A given space is to be maintained at 78°F dry-bulb and 65°F wet-bulb. The total heat gain to a zone  $Q_t$  is 60,000 Btu/hr of which 42,000 Btu/hr is sensible heat transfer (i.e., SHR = 0.70). The

outdoor air requirement of the occupants totals 500 cfm, and the outdoor air condition is 90°F dry-bulb and 55% RH.

Determine:

- a. the quantity and state of the supply air to the space
- b. required capacity of the cooling and dehumidification equipment

Assumptions: Neglect the heat added by the supply fan. Use the notation followed in Figure 2.2.13.

The problem is under-specified since neither the supply air flow rate nor the supply air temperature are specified. Usually one of these needs to be known in order to solve the problem as stated above. A typical rule of thumb is that the supply air dry-bulb temperature entering the space is to be 20°F cooler than the room temperature. Using this rule,  $T_{1db} = 58°F$ .

The following exercise shows the steps that an HVAC design engineer would undertake to size HVAC equipment.

Start with determining relevant properties of states that are fully specified.

Outdoor air: 
$$T_{4db} = 90^{\circ}F$$
,  $RH_4 = 55\%$ . Then  $v_4 = 14.23$  ft<sup>3</sup>/lba,  $W_4 = 0.0170$  lbw/lba, and  $h_4 = 40.8$  Btu/lba

Return or room air:  $T_{2db} = 78^{\circ}F$ ,  $T_{2wb} = 65^{\circ}F$ . Then  $W_2 = 0.0102$  lbw/lba and  $h_2 = 30$  Btu/lba

Also,  $T_6 = T_1$  because we ignore the fan temperature rise.

The supply air mass flow is determined from a sensible heat balance. (Note that an enthalpy or total heat balance cannot be used since the humidity of the supply air is unknown.)

Then 
$$m_{a1} = \frac{Q_s}{c_p(T_2 - T_1)} = \frac{42,000}{0.245(78 - 58)} = 8571 \text{ lba/hr}$$

First, we use the total load to determine the enthalpy of the room supply air:

$$h_1 = -\frac{Q_t}{m_{a1}} + h_2 = -\frac{60,000}{8571} + 30 = 23$$
 Btu/lba

Second, we analyze the adiabatic mixing process at the outdoor air intake. The mass flow rate of outdoor air intake  $m_{a4} = \frac{cfm}{v_4} = \frac{500 \times 60 \text{ min/hr}}{14.23} = 2110 \text{ lba/hr}$ 

Hence, the amount of recirculated air:

$$m_{a2} = 8571 - 2110 = 6461 \text{ lba/hr}$$

An energy balance at the mixing point yields

$$h_5 = \frac{m_{a4}h_4 + m_{a2}h_2}{m_{a1}} = \frac{2110 \times 40.8 + 6461 \times 30}{8571} = 32.6 \text{ Btu/lba}$$

Similarly, humidity and temperature balances yield

$$W_5 = 0.01191$$
 lbw/lba, and

$$T_{5db} = 82^{\circ}F.$$

Finally, the total cooling capacity of the cooling coil can be determined:

 $Q_c = m_{al}(h_1 - h_5) = 8571(23 - 32.6) = -82,282$  Btu/hr = 6.86 cooling tons

How much of this cooling load is sensible can also be deduced as follows:

$$Q_{sc} = m_{a1} \cdot c_{p} \cdot (T_5 - T_1) = 8571 \times 0.245(58 - 32) = -50,397 \text{ Btu/hr}$$

Similar to the room SHR, a cooling coil SHR can also be defined. For this system:

$$SHR_c \equiv \frac{Q_{sc}}{Q_c} = \frac{50,397}{82,282} = 0.612$$
.

Note that this is different from the room SHR = 0.7.

At this point, coils and fans can be specified by the designer. If heating is required, similar heating load and sizing calculations are needed.

# 2.2.6 Human Comfort

Proper HVAC system design and operation requires simple objective criteria that ensure comfort by the majority of occupants.

Human indoor comfort can be characterized by to the occupants' feeling of well-being in the indoor environment. It depends on several interrelated and complex phenomena involving subjective as well as objective criteria. Categories of factors include personal (such as metabolism and clothing), measurable (environmental parameters), and psychological (such as color, sound, light). There is an increasing realization that work productivity suffers if indoor comfort is compromised. Hence, providing for proper indoor comfort is acquiring greater importance than in the past. Because of individual differences, it is impossible to specify a thermal environment that will satisfy everyone. ASHRAE Standard 55-1992 specifies criteria for the thermal environment that is acceptable to at least 80% of the occupants. These criteria are described below.

## Thermal Balance of the Human Body

The body is a heat engine. It converts chemical energy of the food consumed into both heat to sustain metabolism and work. The harder the body exercises or works, the greater the need to reject heat in order for the body to maintain thermal balance. The human body primarily rejects heat to the environment from the body surface by convection, radiation, or evaporation.

The total energy production rate of the body can be written as

$$Q + W = M A_{sk} \tag{2.2.22}$$

where Q = heat production rate

W =rate of work

M = metabolic rate

 $A_{sk}$  = total surface area of skin.

The metabolic rate is customarily expressed in units of mets (or M) where

$$1 \text{ M} = 1 \text{ met} = 18.4 \text{ Btu/h ft}^2 = 58.2 \text{ W/m}^2$$

Since the area  $A_{sk}$  for adults is of the order of 16 to 22 ft<sup>2</sup> (1.5 to 2 m<sup>2</sup>), heat production rates by adults are about 340 Btu/h (110W) for typical indoor activities. Metabolic rates in units of mets for various activities are shown in Table 2.2.4.

Analysis of the rate of heat produced in the body is based on a steady state energy balance. Because the body maintains the interior body temperature at a fairly constant value (about 98.2°F or 36.8°C) by controlling the perspiration rate and blood flow, a steady state model suggested by Fanger (1970) is often

	Btu/hr $\cdot$ ft <sup>2</sup>	$W/m^2$	met <sup>a</sup>
Resting			
Sleeping	13	40	0.7
Reclining	15	45	0.8
Seated, quiet	18	60	1.0
Standing, relaxed	22	70	1.2
Walking (on level surface)			
2.9 ft/s (0.88 m/s)	37	115	2.0
4.4 ft/s (1.3 m/s)	48	150	2.6
5.9 ft/s (1.8 m/s)	70	220	3.8
Office activities			
Reading, seated	18	55	1.0
Writing	18	60	1.0
Typing	20	65	1.1
Filing, seated	22	70	1.2
Filing, standing	26	80	1.4
Walking about	31	100	1.7
Lifting, packing	39	120	2.1
Driving/flying			
Car driving	18-37	60-115	1.0-2.0
Aircraft, routine	22	70	1.2
Aircraft, instrument landing	33	105	1.8
Aircraft, combat	44	140	2.4
Heavy vehicle	59	185	3.2
Miscellaneous occupational activities			
Cooking	29-37	95-115	1.6-2.0
Housecleaning	37-63	115-200	2.0-3.4
Seated, heavy limb movement	41	130	2.2
Machine work			
sawing (table saw)	33	105	1.8
light (electrical industry)	37-44	115-140	2.0-2.4
heavy	74	235	4.0
Handling 110-lb (50-kg) bags	74	235	4.0
Pick-and-shovel work	74-88	235-280	4.0 - 4.8
Miscellaneous leisure activities			
Social dancing	44-81	140-255	2.4-4.4
Calisthenics/exercise	55-74	175-235	3.0-4.0
Tennis, singles	66–74	210-270	3.6-4.0
Basketball	90-140	290-440	5.0-7.6
Wrestling, competitive	130-160	410-505	7.0-8.7

TABLE 2.2.4 Typical Metabolic Heat Generation for Various Activities

Source: Adapted from ASHRAE Fundamentals 1993, p. 8.7.

<sup>a</sup> 1 met = 18.43 Btu/hr  $\cdot$  ft<sup>2</sup> (58.2 W/m<sup>2</sup>).

used. Its basis is that the heat produced in the body is equal to the total amount of heat rejected by the body to its environment.

$$\dot{Q} = \dot{Q}_{con} + \dot{Q}_{rad} + \dot{Q}_{evap} + \dot{Q}_{res,sens} + \dot{Q}_{res,lat}$$
(2.2.23)

where the first three terms refer to the convection, radiation, and evaporation from the skin, and the last two terms to sensible and latent heat of respiration.

Clothing adds thermal resistance to heat flowing from the skin and must be included in the model. The *insulating value of clothing* is measured in units of *clo*, defined as

$$1 \text{ clo} = 0.88 \text{ ft}^2 \text{ h }^\circ\text{F/Btu } (0.155 \text{ m}^2 \text{ K/W})$$
(2.2.24)

	$I_{d}$	$I_T$	
Ensemble Description	(clo)	(clo)	$A_{cl}/A_{sk}$
Walking shorts, short-sleeve shirt	0.36	1.02	1.10
Trousers, short-sleeve shirt	0.57	1.20	1.15
Trousers, long-sleeve shirt	0.61	1.21	1.20
Same as above plus suit jacket	0.96	1.54	1.23
Same as above plus vest and t-shirt	1.14	1.69	1.32
Trousers, long-sleeve shirt,			
long-sleeve sweater, t-shirt	1.01	1.56	1.28
Same as above plus suit jacket			
and long underwear bottoms	1.30	1.83	1.33
Sweat pants, sweat shirt	0.74	1.35	1.19
Long-sleeve pajama top, long pajama trousers,			
short 3/4 sleeve robe, slippers, no socks	0.96	1.50	1.32
Knee-length skirt, short-sleeve shirt,			
pantyhose, sandals	0.54	1.10	1.26
Knee-length skirt, long-sleeve shirt,			
full slip, pantyhose	0.67	1.22	1.29
Knee-length skirt, long-sleeve shirt,			
half slip, pantyhose, long-sleeve sweater	1.10	1.59	1.46
Same as above, replace sweater			
with suit jacket	1.04	1.60	1.30
Ankle-length skirt, long-sleeve shirt,			
suit jacket, pantyhose	1.10	1.59	1.46
Long-sleeve coveralls, t-shirt	0.72	1.30	1.23
Overalls, long-sleeve shirt, t-shirt	0.89	1.46	1.27
Insulated coveralls, long-sleeve thermal			
underwear, long underwear bottoms	1.37	1.94	1.26

 TABLE 2.2.5
 Typical Insulation and Permeability Values for Clothing

 Ensembles
 Figure 1

*Source:* Adapted from *ASHRAE Fundamentals*. With permission. *Note:*  $clo = 0.88 \ ft^2 \cdot {}^{\circ}F/Btu \ (0.155 \ m^2 \ K/w)$ 

The unit of clo is based on the insulating value of the typical American man's business suit in 1941. Table 2.2.5 gives values of the thermal resistance of various clothing ensembles. The values for the clothing alone are given the symbol  $I_{cl}$ , and those for the total thermal resistance between the skin and the environment are given the symbol  $I_T$ . The outside surface area of the clothing should be used rather than skin area when computing heat losses. The surface area ratio between the total surface area of the clothing and the skin area  $A_{cl}/A_{sk}$  is also given in Table 2.2.5.

The convective transfer can be written as

$$\dot{Q}_{con} = A_{cl} h_{con} (T_{cl} - T_a)$$
 (2.2.25)

#### where

A<sub>cl</sub> is the surface area of the clothing and skin in contact with the air,

T<sub>d</sub> is the mean temperature of the clothing and skin in contact with the air,

 $\boldsymbol{h}_{\text{con}}$  is the average convective heat transfer coefficient from clothing, and

T<sub>a</sub> is the dry-bulb temperature of the surrounding air.

The following correlations can be used to determine  $h_{\text{con}}$  in units of Btu/hr ft² °F:

(a) for a seated person:

$h_{con} = 0.55$	$0 \le V \le 40$ ft/min	
$= 0.061 \cdot V^{0.6}$	$40 \le V \le 800$ ft/min	(2.2.26a)

(b) for an active person in still air

$$h_{cov} = (M - 0.85)^{0.39}$$
  $1.1 \le M \le 30$  (2.2.26b)

where M is the metabolic rate in units of met.

The radiative process is more complicated because different surfaces of the environment (or room) may have different temperatures. For example, during winter, a window exposed to the outside may be at a much lower temperature than that of other surfaces. In summer, a sunlit interior wall may be several degrees warmer than an unlit one. Because these differences are usually small, a linearized radiative heat loss can be assumed without much inaccuracy. The emissivities of various indoor surfaces are close to 0.9 and we assume that the surfaces are black. A *mean radiant temperature*  $T_r$  of the environment is defined such that

$$\sigma \sum_{n} F_{cl-n} (T_{cl}^{4} - T_{n}^{4}) = \sigma (T_{cl}^{4} - T_{r}^{4})$$
(2.2.27)

where

the sum includes all surfaces with which the body can exchange direct radiation,

 $F_{cl\mbox{-}n}$  is the radiation view factor from the body to the  $n^{th}$  surrounding surface, and

 $\sigma$  is the Stefan–Boltzman constant.

In other words,  $T_r$  is the temperature of an imaginary isothermal enclosure with which a human body would exchange the same amount of radiation as with the actual environment.

The radiative heat loss from the body can now be written as

$$\dot{Q}_{rad} = A_{cl}h_{rad} (T_{cl} - T_r)$$
 (2.2.28)

where  $h_{rad}$  is the linearized radiative heat transfer coefficient. A numerical value of 0.83 Btu/hr ft<sup>2</sup> °F (4.7 W/m<sup>2</sup> K) is advocated for this coefficient for normal nonmetallic clothing.

We can combine the convective and radiative coefficients into one single total heat transfer coefficient:

$$\mathbf{h}_{c+r} \equiv \mathbf{h}_{con} + \mathbf{h}_{rad} \tag{2.2.29}$$

Then, Equations 2.2.25 and 2.2.28 can be written together as:

$$\dot{Q}_{con} + \dot{Q}_{rad} = A_{cl} \cdot h_{c+r} \cdot (T_{cl} - T_{op})$$
(2.2.30)

where  $T_{op}$  is called the *operative temperature* defined as

$$T_{\rm op} \equiv \frac{h_{\rm con} T_{\rm a} + h_{\rm rad} T_{\rm r}}{h_{\rm c+r}}$$
(2.2.31)

Often  $T_{op}$  is close to the simple arithmetic average of  $T_a$  and  $T_r$ . The comfort indices above do not include humidity effects. One can include the evaporative skin loss in the above treatment by defining the *adiabatic equivalent temperature* which is a linear combination of  $T_{op}$  and the vapor pressure of the air, which depends on skin wetness and on clothing permeability. This quantity would then allow combining environmental dry-bulb temperature and humidity as well as the surrounding surface temperatures into a single temperature index that completely determines the total heat loss from the skin. It is used as the basis of the ASHRAE comfort chart described below.

Another temperature index, the *effective temperature*  $ET^*$ , is also used in the analysis of thermal comfort. Like the adiabatic equivalent temperature, it is a linear combination of  $T_{op}$  and the vapor pressure. More strictly, it is the temperature of an isothermal black enclosure with 50% RH where the

body surface would experience the same heat loss as in the actual space. The 50% level is chosen as the reference value because it is the most common and widely accepted level of indoor humidity.

**Example 8**: Determine the operative temperature of an enclosure whose dry-bulb temperature is 75°F and mean radiant temperature is 80°F, and where the occupants walk around at 4.4 ft/s.

From Table 2.2.4, the activity level corresponds to a met level of 2.6. The convective heat transfer coefficient  $h_{con}$  is determined from Equation 2.2.26b:

$$h_{con} = (2.6 - 0.85)^{0.39} = 1.244 \text{ Btu/hr} \cdot \text{ft}^2 \cdot \text{°F}$$

We use the standard value for the radiative heat-transfer coefficient of 0.83 Btu/hr ft<sup>2</sup> °F. Thus from Equation 2.2.31,

$$T_{op} = \frac{1.244 \times 75 + 0.83 \times 80}{1.244 + 0.83} = 77.0^{\circ}F$$

#### **Conditions for Thermal Comfort**

The environmental parameters discussed above are the primary factors used to characterize human comfort. There are also secondary effects such as nonuniformity of the environment, visual stimuli, age, and outdoor climate (ASHRAE, 2001). Studies on 1600 college-age students revealed correlations between comfort level, temperature, humidity, sex, and length of exposure. Several trends are described below.

The thermal sensation scale adopted for voting by the students is called the ASHRAE thermal sensation scale, and is represented as follows:

- +3 hot
- +2 warm
- +1 slightly warm
- 0 neutral
- -1 slightly cool
- -2 cool
- -3 cold

The relevant index of acceptability of the indoor environment is the predicted mean vote (PMV) for which an empirical correlation has been developed:

$$PMV = a^*t + b^*P_v + c^* \tag{2.2.32}$$

where the numerical values of the coefficients a\*, b\*, and c\* are given in Table 2.2.6.

In general, the distribution of votes will always show considerable scatter. A useful index of acceptability of an environment is the percentage of people dissatisfied (PPD), defined as people voting outside the range of -1 to +1. When the PPD is plotted versus the mean vote of a large group characterized by the PMV, one typically finds a distribution such as shown in Figure 2.2.16 This graph shows that even under optimal conditions (i.e., a mean vote of zero), at least 5% of occupants are dissatisfied with the thermal comfort.

ASHRAE Standard 55 specifies boundaries of the comfort zones where 80% of occupants are thermally comfortable. The specification of these zones is based on the concept of ET\* as depicted in Figure 2.2.17. This chart is similar to the psychrometric chart, but the abscissa is the operative temperature  $T_{op}$  rather than the dry-bulb temperature. The acceptable values of temperature and humidity are indicated by the shaded zones in Figure 2.2.17. Numerical values for the comfort zone are are provided in Table 2.2.7. In winter, an effective temperature of 70 to 72°F (21 to 22°C) is optimum for normally clothed people. In summer, about 76°C would be optimum. The further one strays from these values, the greater the percentage of people who are likely to be dissatisfied. The chart shows different zones for winter and for summer because comfort depends on the insulation value of the clothing, as discussed earlier.

		Coefficients					
Exposure Period,	Sex	English Units, t (°F), P <sub>v</sub> (psia)			SI Units, $t$ (°C), $P_v$ (kPa)		
hr		a*	$b^{\star}$	с*	a*	$b^*$	с*
1.0	Male	0.122	1.61	-9.584	0.220	0.233	-5.673
1.0	Female	0.151	1.71	-12.080	0.272	0.248	-7.245
1.0	Combined	0.136	1.71	-10.880	0.245	0.248	-6.475
3.0	Male	0.118	2.02	-9.718	0.212	0.293	-5.949
3.0	Female	0.153	1.76	-13.511	0.275	0.255	-8.622
3.0	Combined	0.135	1.92	-11.122	0.243	0.278	-6.802

**TABLE 2.2.6** Coefficients  $a^*$ ,  $b^*$ , and  $c^*$  Used to Calculate Predicted Mean Vote

Source: Adapted from ASHRAE Fundamentals 1993, p. 8.16.

*Note:* For young adult subjects with sedentary activity and wearing clothing with a thermal resistance of approximately 0.5 clo,  $t_r = t$ , air velocities  $\leq 40$  ft/min (0.2 m/s).



**FIGURE 2.2.16** Predicted percentage of dissatisfied (PPD) as a function of predicted mean vote (PMV). [Reprinted with permission from *ASHRAE Fundamentals 1993* (IP & SI), p. 8.18.]

**TABLE 2.2.7** Optimum and Acceptable Ranges of Operative Temperature for People during<br/>Light, Primarily Sedentary Activity ( $\leq 1.2 \text{ met}$ ) at 50% Relative Humidity and Mean Air Speed<br/> $\leq 0.15 \text{ m/s} (30 \text{ fpm})$ 

Season	Description of typical clothing	$I_{cl}$ (clo)	Optimum operative temperature	Operative temperature range (10% dissatisfaction criterion)
Winter	heavy slacks, long-sleeve shirt and sweater	0.9	22°C 71°F	20–23.5°C 68–75°F
Summer	light slacks and short-sleeve shirt	0.5	24.5°C 76°F	23–26°C 73–79°F
	minimal	0.05	27°C 81°F	26–29°C 79–84°F



**FIGURE 2.2.17** Acceptable ranges of operative temperature and humidity for people in typical summer and winter clothing during light, primarily sedentary activity ( $\leq$  1.2 met). The ranges are based on a 10% dissatisfaction criterion.

The left and right boundaries of the shaded zones are lines of constant adiabatic equivalent temperature, hence lines of constant heat loss. They are sloped from upper left to lower right because the evaporative heat loss from the body decreases as the humidity ratio of the air increases.

The comfort chart is applicable to reasonably still air (velocities less than 30 ft/min or 0.15 m/s), with occupants seated at rest or doing light work (met = 1.2) and to spaces where enclosing surfaces are at the mean temperature equal to the dry-bulb temperature. When the activity level is between 1.2 and 3 mets, Standard 55-1992 recommends that the operative temperature be determined as:

$$\begin{split} T_{op} &= T_{op,std} - 5.4 \ (1.0 + clo)(M - 1.2) \ ^{o}F \\ T_{op} &= T_{op,std} - 3.0 \ (1.0 + clo)(M - 1.2) \ K \end{split} \tag{2.2.33}$$

where clo=clothing insulation, and the subscript std designates the value below 1.2 met.

The ASHRAE Comfort chart applies to clothing levels of clo=0.9 during winter and clo=0.5 during summer. The relationship between clothing insulation and operative temperature so as to be within the ASHRAE 80% acceptability limits is shown in Figure 2.2.18. As the operative temperature increases, one can still maintain comfort by reducing the clothing insulation value, i.e., by dressing lightly.

Within the thermally acceptable temperature ranges stated earlier, there is no minimum air movement that is necessary for thermal comfort. The temperature may be increased above these levels provided one can increase the air speed. An estimate is provided by Figure 2.2.19 which corresponds to clothing and activities pertinent to summer comfort zone. However, temperature should not be increased more than 5.4°F (3°C) above the comfort zone and air speeds should be kept less than 160 ft/min (0.8 m/s). Studies have shown that women of all age groups prefer an effective temperature about one degree higher than that preferred by men, while both men and women over 40 years prefer an effective temperature about one degree higher than that desired by younger people.

Finally, thermal stratification in the room also affects comfort. Vertical air temperature difference should not exceed 5°F (3 K) between 4 in (0.1 m) and 67 in (1.7 m) above the floor. Other specific recommendations are provided in ASHRAE Standard 55-1992.



**FIGURE 2.2.18** The recommended range of clothing insulation providing acceptable thermal conditions at a given operative temperature for people during light, primarily sedentary activity ( $\leq$ 1.2 met). The limits are based on a 10% dissatisfaction criterion.



**FIGURE 2.2.19** Air speed required to offset increased temperature. The air speed increases in the amount necessary to maintain the same total heat transfer from the skin. This figure applies to increases in temperature above those allowed in the summer comfort zone with both  $\bar{t}_r$  and  $t_a$  increasing equally. The starting point of the curves at 0.2 m/s (40 fpm) corresponds to the recommended air speed limit for the summer comfort zone at 26°C (79°F) and typical ventillation (i.e., turbulence intensity between 30% and 60%). Acceptance of the increased air speed requires occupant control of the local air speed.

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